Uncertainty Analysis Using ISG's Uncertainty Analyzer 3.0

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23rd Transducer Workshop

Buffalo, New York

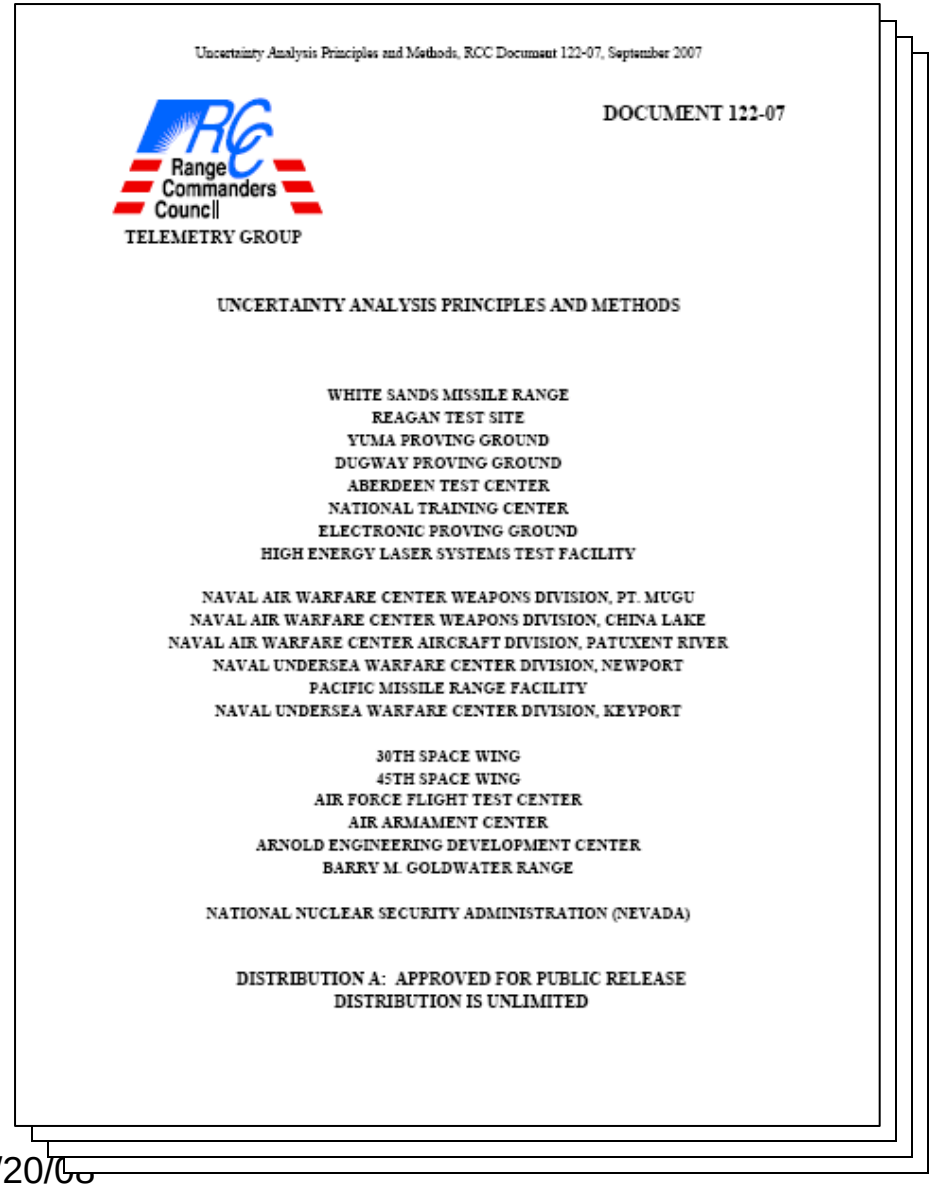
Wednesday, June 18, 2008

Purpose of this Tutorial

This tutorial will introduce the methodology used to perform a measurement uncertainty analysis. The considerations needed for each step will be illustrated in an example analysis.

Uncertainty Analyzer 3.0 is a software tool used in this process. This tutorial will show how this tool is utilized in the process, but it is not meant to be comprehensive training on how to use the software.

IRIG-122-07 Uncertainty Analysis Principles and Methods



Uncertainty Analysis Principles and Methods, RCC Document 122-07, September 2007

DOCUMENT 122-07

Range Commanders Council

TELEMETRY GROUP

UNCERTAINTY ANALYSIS PRINCIPLES AND METHODS

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Integrated Sciences Group (ISG) authored the RCC Telemetry Group's IRIG-122-07 Uncertainty Analysis Principles and Methods document.

This document provides information about the mathematical concepts and methods used to estimate measurement uncertainty.

Uncertainty Analyzer 3.0 executes the mathematics described in this document.

Uncertainty Analyzer 3.0

Uncertainty Analyzer 3.0 was developed by Integrated Sciences Group in Bakersfield, CA.

The software is a tool for conducting and reporting uncertainty analyses for direct and multivariate measurements and measurement systems.

The software performs all of the mathematics and statistics in the background. However, it does not provide any of the technical expertise of the measurement system – that is left to the user of the software.

INTEGRATED SCIENCES GROUP

State-of-the-Art Measurement Analysis Software, Training and Consulting Services

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Address http://www.isgmax.com/

UncertaintyAnalyzer 3.0 Summary Info

Additional Information

Program Features

Software Comparison

Purchasing Options

Why Upgrade to 3.0?

Service Updates

References

Training Options

UncertaintyAnalyzer ($995 USD) is the world's most powerful and versatile measurement uncertainty analysis software product available. With an exhaustive list of features, capabilities and on-screen information, UncertaintyAnalyzer is a complete, state-of-the-art tool for conducting and reporting uncertainty analyses for direct measurements, multivariate measurements and measurement systems. Our customers consider it to be the Swiss Army Knife of uncertainty analysis software products.

UncertaintyAnalyzer incorporates and extends the concepts and methods found in the *ISO Guide to the Expression of Uncertainty in Measurement* (the ISO GUM). Consequently, UncertaintyAnalyzer greatly facilitates compliance with ISO 17025 and ANSI/NCSL Z540.3.

UncertaintyAnalyzer's built-in statistical analysis methodologies are combined with user technical expertise through a user-friendly graphical interface. All math and statistics are performed in background, requiring little or no statistical training.

Since just about everyone involved in making or interpreting measurements needs to analyze measurement uncertainty, this product is a must for the professional toolbox of test engineers, calibration engineers, research scientists, and the general technical community.

UncertaintyAnalyzer 3.0 runs on Windows 95, 98, 2000, ME, NT and XP.

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Result of an Uncertainty Analysis

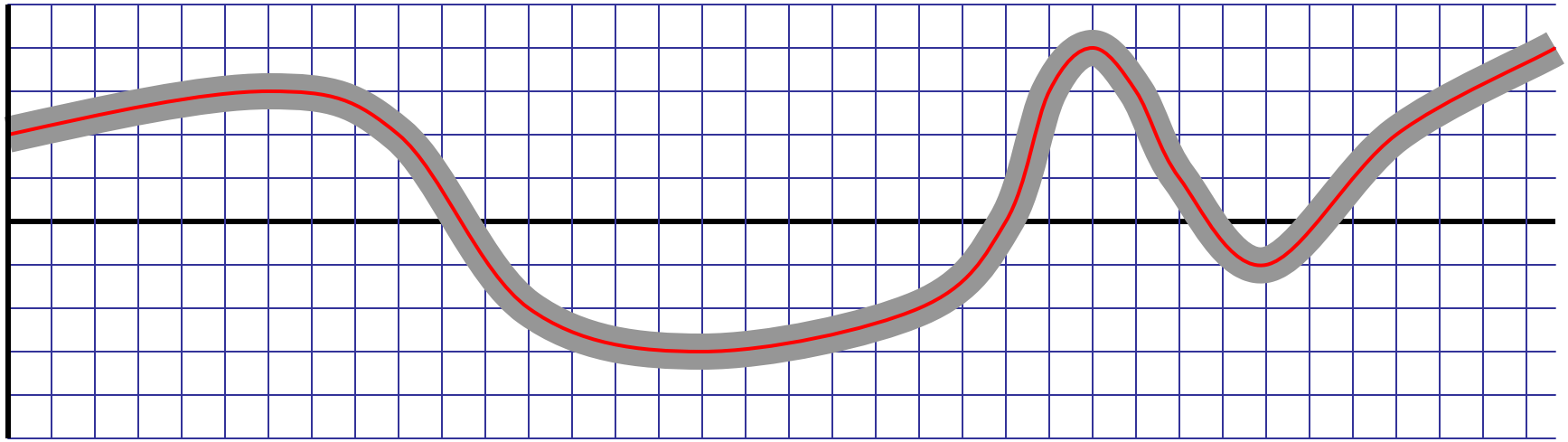
The result of an uncertainty analysis defines the boundaries of the measurement error (ex. ±0.31 G's). One misconception is that an uncertainty analysis tells you the error in a measurement.

$$X_{meas} = X_{true} + X_{error}$$

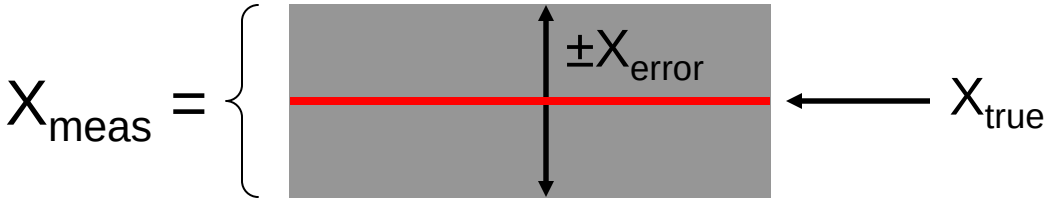
If that were the case, simply subtracting the error from the measurement, would yield the true value.

However, the measurement error (X_{error}) is a random variable which is bounded and we have a certain confidence that it falls within that bound. Unfortunately for those analyzing data, the true value (X_{true}) of a measurement is never known.

Graphically Showing x_{meas}, x_{true}, and x_{error}



We do not know what the true value (shown in red) is, but we do know with some level of confidence that it falls within a certain error bound (shown in gray).

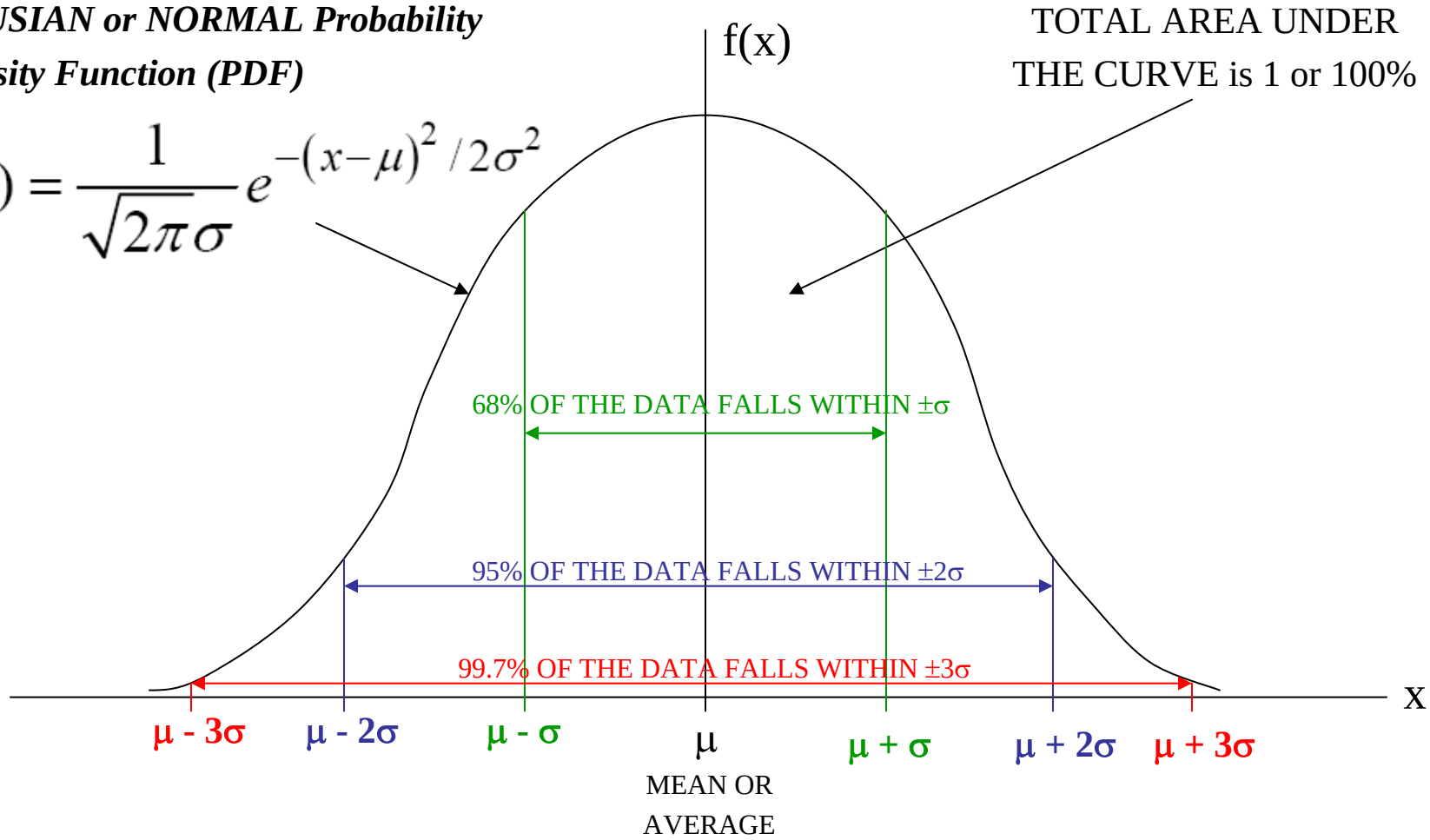


The Normal Distribution

The confidence level is set on boundaries of multiple standard deviations from the mean. For the Normal Distribution, the confidence levels are based on the area under the curve, which has the shape shown below.

GAUSIAN or NORMAL Probability Density Function (PDF)

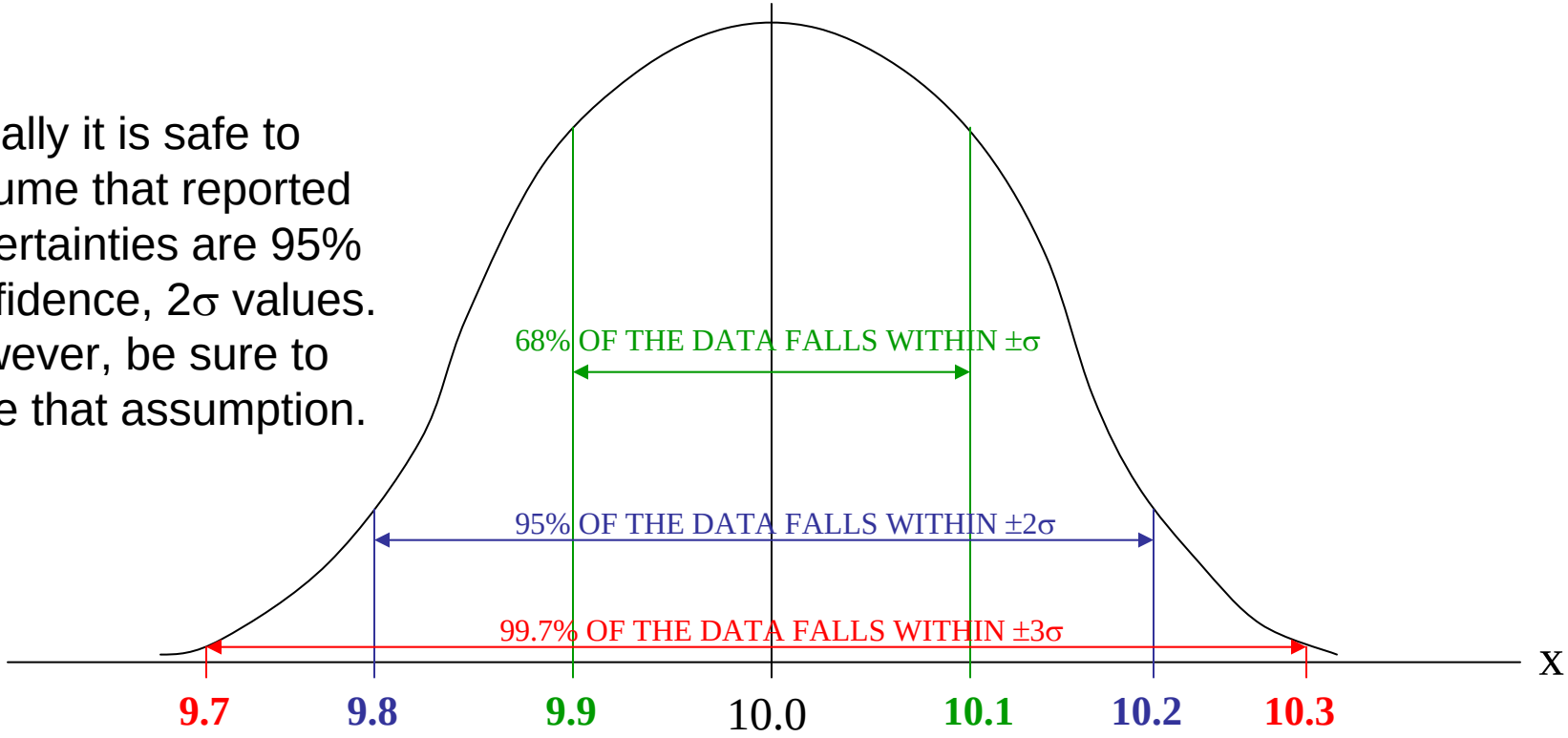
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$$



How to Interpret the Reported Uncertainty

If the expected value (true value) of a measurement is 10, and the 1σ number (which is one standard deviation) is calculated to be 0.1 G, then the uncertainty is usually reported to be ±2σ or ±0.2 G. This can be interpreted as the measured value having a 95% likelihood of being within ±0.2 G's of the true value. Or in other words, if the true value is 10 G's, the measured output has a 95% confidence of being in the range of 9.8 to 10.2 G's.

Usually it is safe to assume that reported uncertainties are 95% confidence, 2σ values. However, be sure to state that assumption.



The uncertainty bounds the errors of a measurement.

Assuming a 95% Confidence

- Transducer manufacturers rarely indicate what the confidence intervals are for their specifications
- It is safe to assume that specifications are 2σ values because if the values were 1σ numbers, only 68% of their transducers would pass the specification of being within $\pm 1\sigma$.
- If ever in doubt, contact the manufacturer, but if you must make an assumption, state that assumption in your uncertainty analysis report.

Uncertainty Analysis Procedure

The executive summary of IRIG-122-07 describes the uncertainty analysis procedure as the following steps:

1) Define the Measurement Process
2) Identify the Error Sources and Distributions
3) Estimate Uncertainties
4) Combine Uncertainties
5) Report the Analysis Results

Step 2 is the most difficult step of the analysis procedure. This will be illustrated in the example analysis.

Example Uncertainty Analysis

- Go through the Uncertainty Analysis Procedure for an accelerometer measurement.
- Create transfer function and define sources of error in the measurement.
- Introduce Uncertainty Analyzer 3.0 as an application tool to handle the mathematical equations described in IRIG-122-07.
- Pareto diagrams will be used to identify large sources of error and determine if they can be reduced.
- Reporting the final analysis.

Acceleration Measurement

The acceleration measurement example will utilize an Endevco 7290A-30 accelerometer with a TTC SCD-108S signal conditioning card. To simplify this analysis, there will not be an end-to-end voltage substitution calibration performed on the system.

The measurement requirements are as follows:

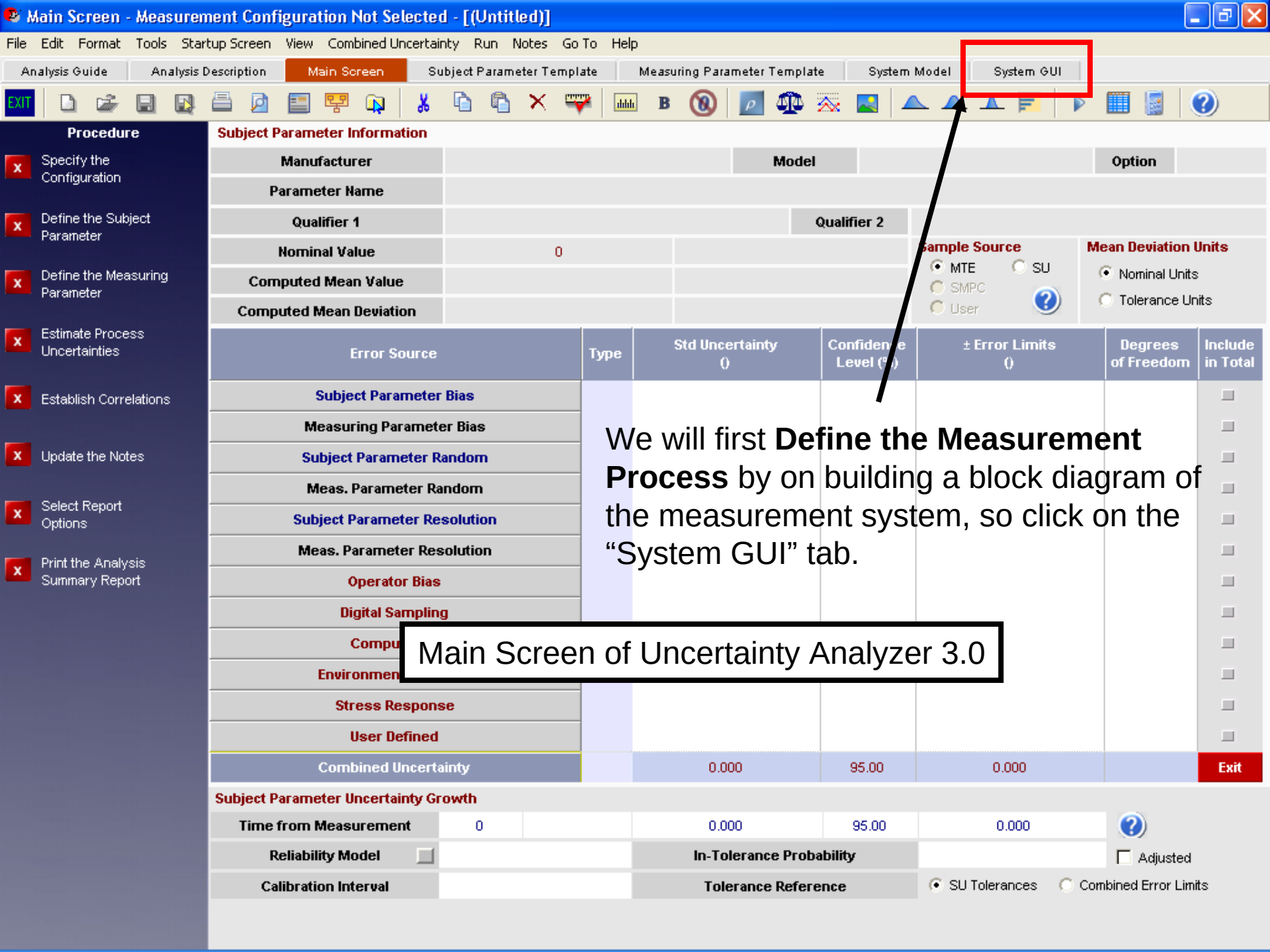
- Normal Acceleration
- EU Range: ±30 G's
- Frequency Range: 0 to 300 Hz

Assumptions

An uncertainty analysis should not just contain a list of numbers, it should also contain text describing how those resulting numbers were obtained.

Because there are a lot of assumptions made in an uncertainty analysis, it is important that those assumptions be documented. Those reading the analysis must understand where the results came from.

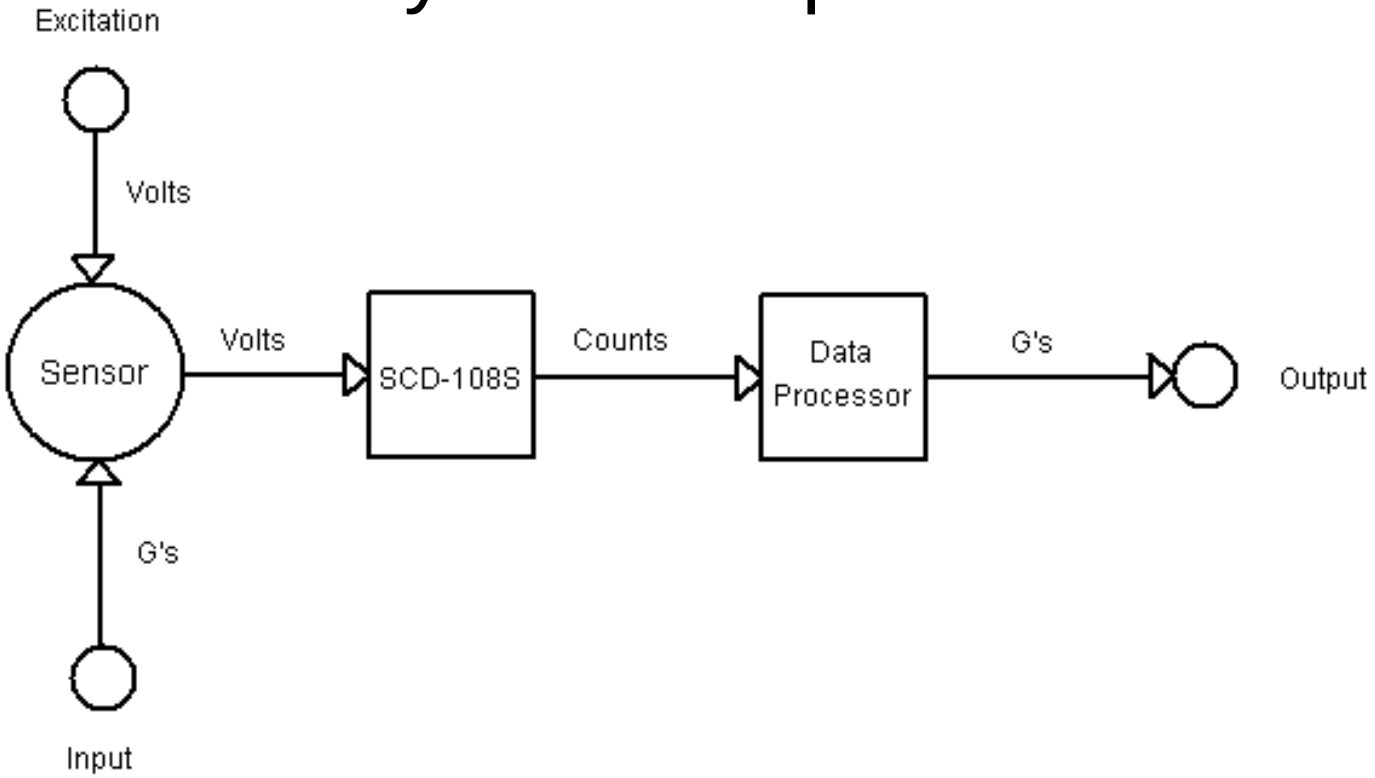
We will now start by running
Uncertainty Analyzer 3.0


Main Screen - Measurement Configuration Not Selected - [(Untitled)]
File Edit Format Tools Startup Screen View Combined Uncertainty Run Notes Go To Help
Analysis Guide
Analysis Description
Main Screen
Subject Parameter Template
Measuring Parameter Template
System Model
System GUI
EXIT
Procedure
Specify the Configuration
Define the Subject Parameter
Define the Measuring Parameter
Estimate Process Uncertainties
Establish Correlations
Update the Notes
Select Report Options
Print the Analysis Summary Report
Subject Parameter Information
Manufacturer
Model
Option
Parameter Name
Qualifier 1
Qualifier 2
Nominal Value
0
Computed Mean Value
Computed Mean Deviation
Sample Source
MTE
SU
SMPC
User
Mean Deviation Units
Nominal Units
Tolerance Units
Error Source
Type
Std Uncertainty ()
Confidence Level (%)
± Error Limits ()
Degrees of Freedom
Include in Total
Subject Parameter Bias
Measuring Parameter Bias
Subject Parameter Random
Meas. Parameter Random
Subject Parameter Resolution
Meas. Parameter Resolution
Operator Bias
Digital Sampling
Stress Response
User Defined
Combined Uncertainty
0.000
95.00
0.000
Exit
Subject Parameter Uncertainty Growth
Time from Measurement
0
0.000
95.00
0.000
Reliability Model
In-Tolerance Probability
Adjusted
Calibration Interval
Tolerance Reference
SU Tolerances
Combined Error Limits
We will first Define the Measurement Process by on building a block diagram of the measurement system, so click on the "System GUI" tab.
Main Screen of Uncertainty Analyzer 3.0

1) Define the Measurement Process

The Block Diagram

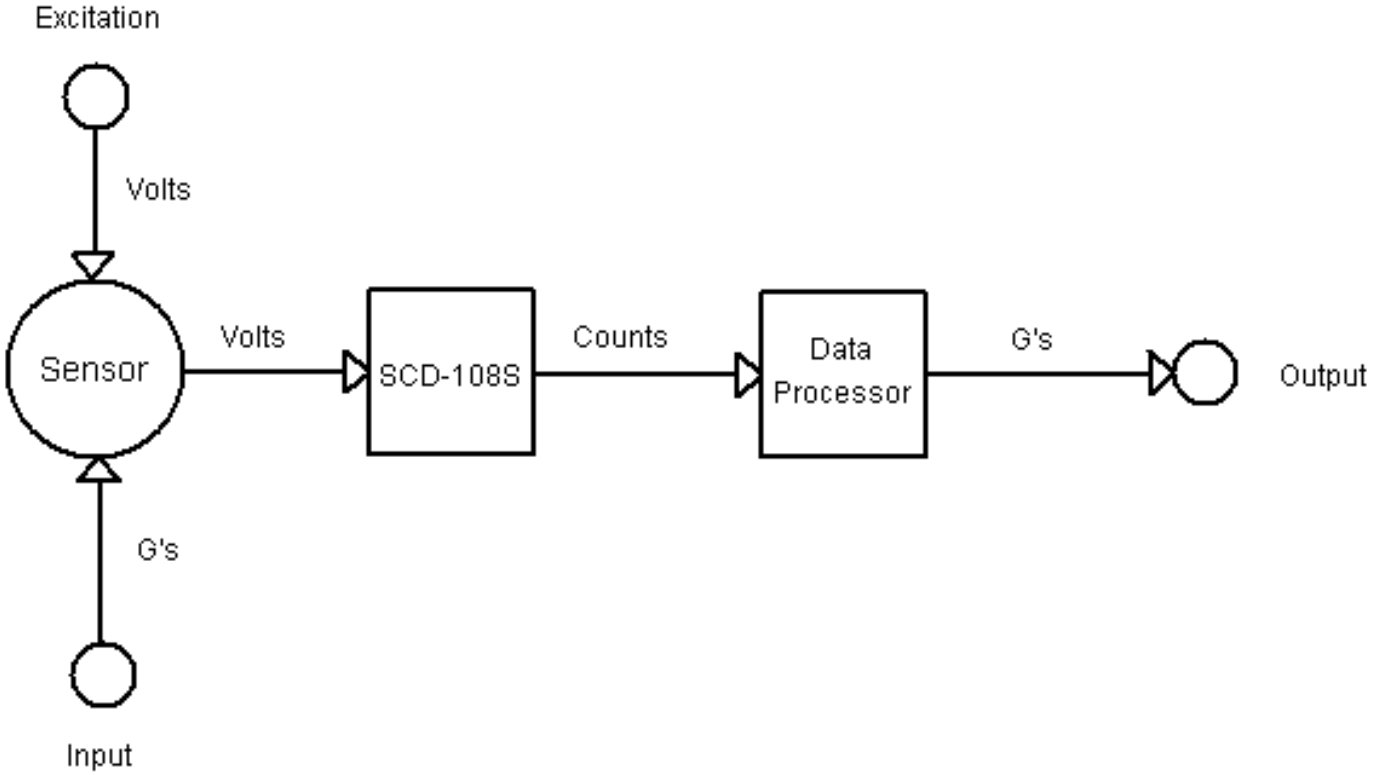
Uncertainty Analyzer 3.0 allows you to create the block diagram in the **System GUI**. This is the first step that must be done before any analysis takes place.

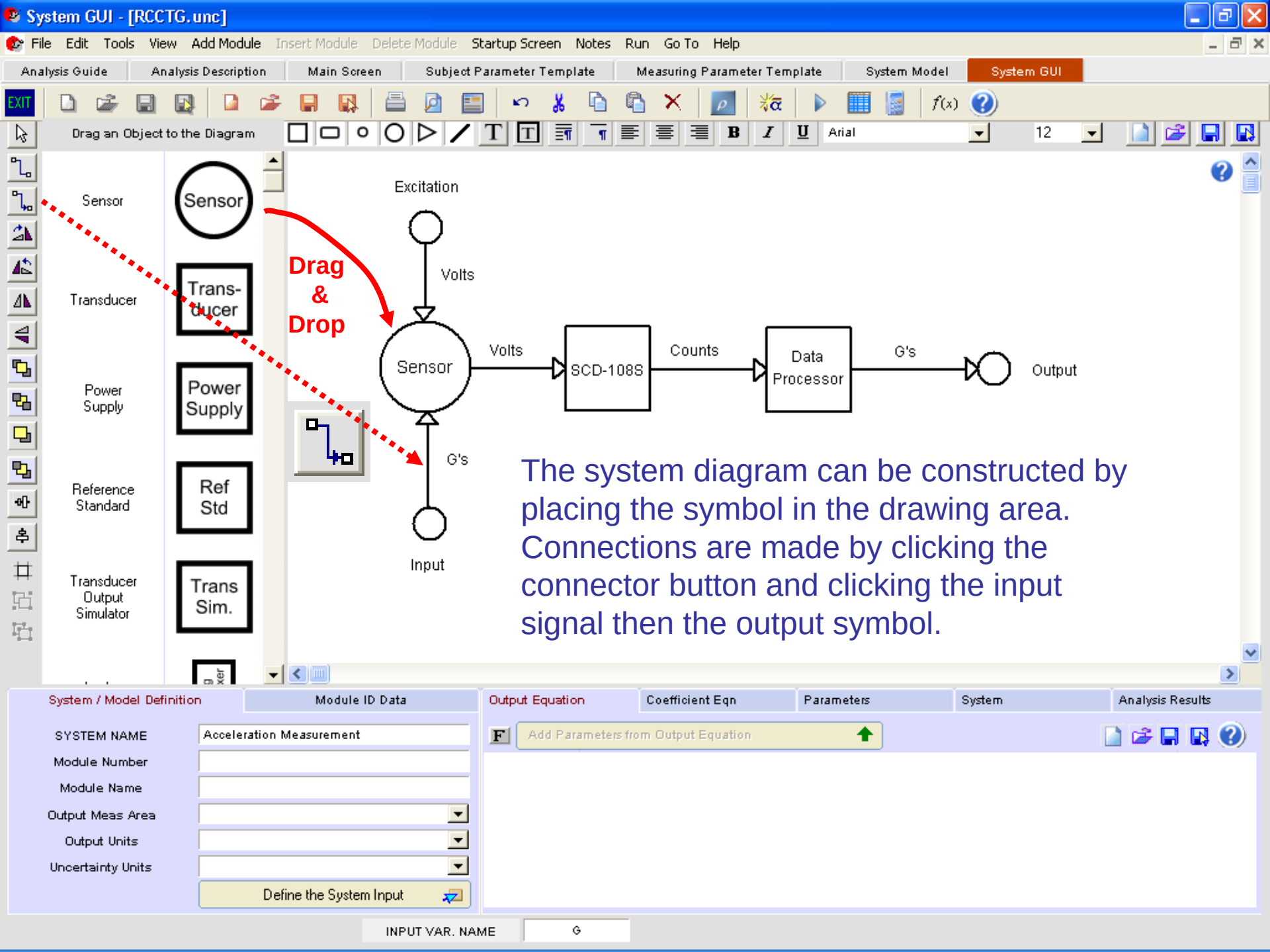


1) Define the Measurement Process

The Block Diagram

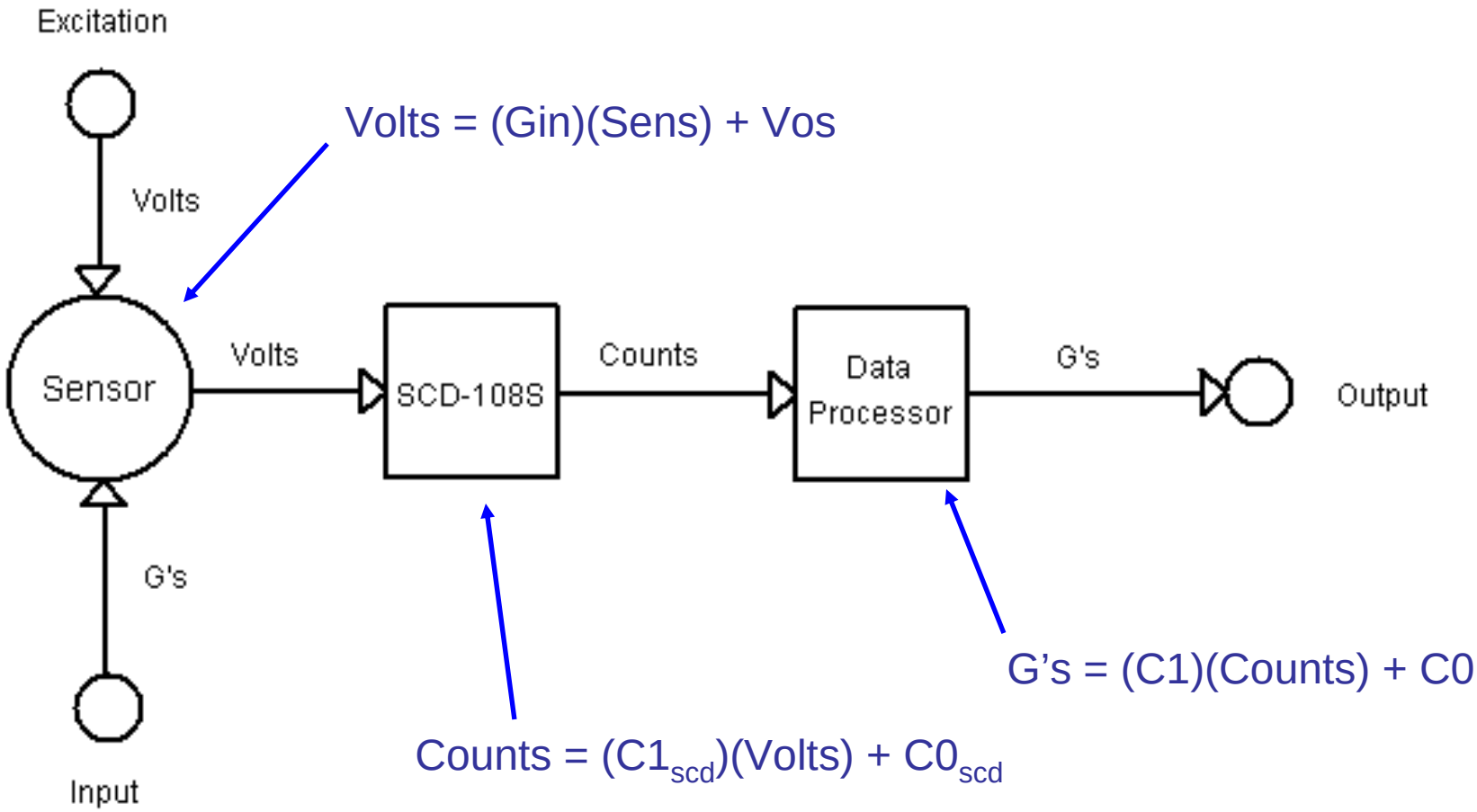
- Give each module a name
- Indicate the inputs and outputs of each module
- Note the engineering units at each input/output




System GUI - [RCCTG.unc]
File Edit Tools View Add Module Insert Module Delete Module Startup Screen Notes Run Go To Help
Analysis Guide
Analysis Description
Main Screen
Subject Parameter Template
Measuring Parameter Template
System Model
System GUI
EXIT
Drag an Object to the Diagram
Arial
12
Sensor
Transducer
Power Supply
Reference Standard
Transducer Output Simulator
Sensor
Trans-ducer
Power Supply
Ref Std
Trans Sim.
Drag & Drop
Excitation
Volts
Sensor
Volts
SCD-108S
Counts
Data Processor
G's
Output
G's
Input
The system diagram can be constructed by placing the symbol in the drawing area. Connections are made by clicking the connector button and clicking the input signal then the output symbol.
System / Model Definition
Module ID Data
Output Equation
Coefficient Eqn
Parameters
System
Analysis Results
SYSTEM NAME
Acceleration Measurement
Module Number
Module Name
Output Meas Area
Output Units
Uncertainty Units
Define the System Input
Add Parameters from Output Equation
INPUT VAR. NAME
G

Determine the Transfer Functions

Next, write down the transfer functions for each of the modules in the diagram. Don't worry about any contributing errors at this point. They will be added later.



System GUI - [RCCTG.unc]

File Edit Tools View Add Module Insert Module Delete Module Startup Screen Notes Run Go To Help

Analysis Guide | Analysis Description | Main Screen | Subject Parameter Template | Measuring Parameter Template | System Model | System GUI

Drag an Object to the Diagram

Sensor — Transducer — Power Supply — Reference Standard — Transducer Output Simulator

Excitation → Volts → Sensor → Volts → SCD-108S → Counts → Data Processor → G's → Output

Sensor → G's → Input

By clicking on the sensor symbol, you can enter the information for the accelerometer.

System / Model Definition | Module ID Data

SYSTEM NAME	Acceleration Measurement
Module Number	1 << First module in the series connection
Module Name	Accelerometer
Output Meas Area	DC Voltage
Output Units	V
Uncertainty Units	V

System / Model Definition | Module ID Data | Output Equation

SYSTEM NAME	Acceleration Measurement
Module Number	1
Module Name	Accelerometer
Output Meas Area	DC Voltage
Output Units	V
Uncertainty Units	V

Define the System Input

Add Parameters from Output Equation

Information will be entered for each module in the system diagram.

INPUT VAR. NAME | G

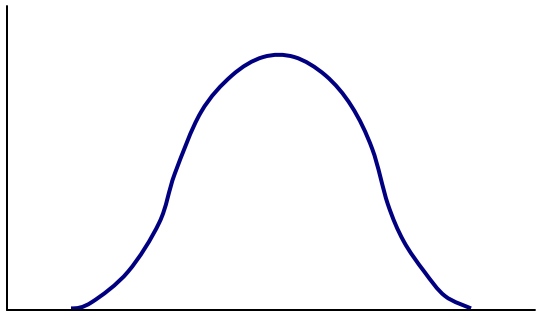
2) Identify the Error Sources and Distributions

Analyze Each of the Modules

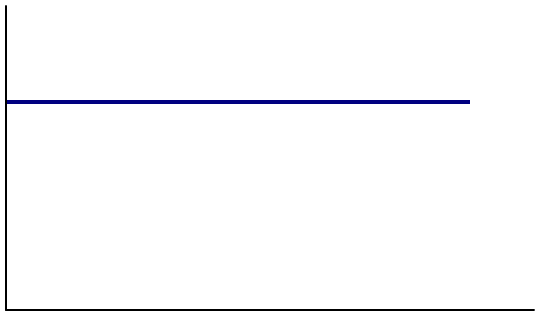
Each module will be studied to determine what contributing errors exist within its transfer function. These equations will then be entered into Uncertainty Analyzer.

Choosing a Distribution for the Defined Errors

Normal distributions are relevant to most real world measurements. In some cases, as in quantization error, there is an equal likelihood that the measurement is above or below a quantization level, Therefore, a uniform distribution will be used.



NORMAL



UNIFORM

Entering the Information in Uncertainty Analyzer 3.0

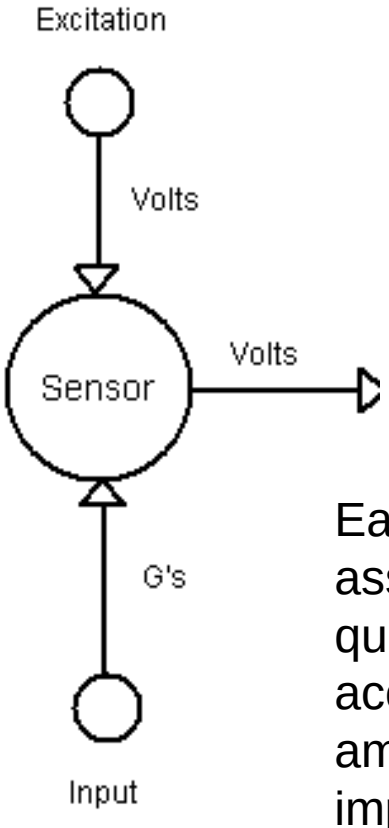
As each transfer function is defined for each module, the equations will be entered into Uncertainty Analyzer.

The information can be entered in the **System GUI** screen (where the block diagram was created) or the **System Model** screen (introduced later).

The variables within the equations are called parameters.

The specification sheets will identify the errors which exist for the defined parameters in the transfer function.

7290A Accelerometer



Volts = Gin x Sens + Vos + Exdcr

Each of the parameters of the transfer function has an associated error that must be accounted for. The errors are quantified in the specification sheet for the 7290A accelerometer. Sometimes, spec sheets can be ambiguous, that is where documenting assumptions is very important.

Unless noted, all errors are assumed to have a 95% bound and be distributed normally.

Entering the Transfer Function

The **Output Equation** tab allows you to enter the transfer function for that module. The equation defining the module output <u>must</u> be entered first.

Red indicates that the parameter has not been defined yet.

System GUI - [RCCTG.unc]

File Edit Tools View Add Module Insert Module Delete Module Startup Screen Notes Run Go To Help

Analysis Guide | Analysis Description | Main Screen | Subject Parameter Template | Measuring Parameter Template | System Model | System GUI

Drag an Object to the Diagram

Sensor | Transducer | Power Supply | Reference Standard | Transducer Output Simulator

Excitation | Volts | Sensor | Volts | Counts | Data | G's | G's | Input

Output Equation | Coefficient Eqn | Parameters

F | Add Parameters from Output Equation

Volts = Gin * Sens + Vos + Exdcr

System / Model Definition | Module ID Data | Output Equation | Coefficient Eqn | Parameters | System | Analysis Results

SYSTEM NAME	Acceleration Measurement
Module Number	1
Module Name	Accelerometer
Output Meas Area	DC Voltage
Output Units	V
Uncertainty Units	V

Define the System Input

F | Add Parameters from Output Equation

Volts = Gin * Sens + Vos + Exdcr

INPUT VAR. NAME | G

Volts = Gin x Sens + Vos + Exdcr

Volts: the voltage output from the accelerometer (V)

Gin: the input acceleration sensed by the transducer (G's)

Sens: the voltage sensitivity of the accelerometer to the input (V/G)

Vos: the output of the accelerometer at a 0 G input (V)

Exdcr: includes any other error sources that did not fit into the above quantities (V)

Quantifying Sources of Error

There are two ways to quantify errors contributing to the system:

Type A Estimates – which are estimates of error that are derived from samples of data. This could be from taking repeat measurements with the transducer. The statistical results of the data will quantify the errors used in the analysis. This method however is very time consuming.

Type B Estimates – are drawn from our experience of the measurement and from the specification sheets (the manufacturers experience). This is how most error sources are quantified in an uncertainty analysis.

Accelerometer Specifications

The best place to identify errors is to consult the specification sheet. For this example, the accelerometer was calibrated in the transducer laboratory. Therefore, some of these specifications and errors (shown in red text) will be replaced by the results of that calibration. If a calibration was not performed, then these quantities in the specification would be used.

Range	**±30 Gpk**
Full Scale Output (FSO)	**4 volts**
Operating Temp Range	-55°C to 121°C
Sensitivity @100Hz	**66 ±4 mV/G**
Sensitivity with Excitation	+0.1%/V Typ
Offset with Excitation	-0.5mV/V Typ
Frequency Response	**±5%**
Mounted Resonance Freq	5500 Hz
Non-Linearity & Hysteresis	±0.20% FSO Typ, ±0.5% FSO Max
Transverse Sensitivity	2% Max
Zero Measurand Output	**±50 mV Max**
Dampening Ratio	0.7
Dampening Ratio Change	+0.08%/°C (-55°C to 121°C)
Thermal Zero Shift	±1.0% FSO Max (0-50°C)
	±2.0% FSO Max (-25-75°C)
Thermal Sensitivity Shift	±2.0% Max (0-50°C)
	±3.0% Max (-25-75°C)
Thermal Transient Error	<0.001 G/°C
Threshold (resolution)	0.008 G's
Base Strain Sensitivity	0.01 G's
Magnetic Susceptibility	<0.1 G's
Residual Noise	100uVrms Typ, 500uVrms Max (0.5 to 100 Hz)
	500uVrms Typ, 1mVrms Max (0.5 to 10 KHz)

Transducer Lab Calibration

The results from the transducer laboratory calibration were the following:

G's	Volts	G's	Volts
-30	-1.945	30	2.030
-24	-1.541	24	1.629
-18	-1.143	18	1.233
-12	-0.747	12	0.838
-6	-0.349	6	0.442
-1	-0.019	1	0.113
0	0.047	0	0.047
1	0.113	-1	-0.019
6	0.442	-6	-0.350
12	0.838	-12	-0.747
18	1.233	-18	-1.143
24	1.629	-24	-1.541
30	2.030	-30	-1.945

LSBF Equations: G's = 15.12179 x Volts -0.6819215

Volts = 0.0661297*G + 0.045095

Transducer Lab Calibration

The sensitivity frequency response was the following:

Freq (Hz)	Sensitivity (mV/G)
0	66.1297 (From LSBF Eq)
10	66.0943
15	66.0630
20	66.0770
30	66.0967
50	66.0720
100	66.1356
159	66.0044
200	65.7564
300	65.9873

Average Sensitivity: 66.0416 mV/G
Sensitivity Std Dev: 0.1109 mV/G
Zero Measured Output: 45.681 mV

Each parameter of the transfer function will be evaluated for error sources listed in the accelerometer's specification.

Identify errors contributing to the sensed G level

Identify errors contributing to sensitivity

$$\text{Volts} = \text{Gin} \times \text{Sens} + \text{Vos} + \text{Exdcr}$$

Identify errors contributing to the offset voltage

Other identified errors which do not fit in the other elements

Volts = Gin x Sens + Vos + Exdcr

Gin is the input acceleration sensed by the transducer plus all of the associated errors. Read through the specification sheet and determine which errors will contribute to the sensing of the acceleration. It is a safe assumption that the listed errors are 2σ (two standard deviation), 95% confidence values.

Range	±30 Gpk
Full Scale Output (FSO)	4 volts
Operating Temp Range	**-55°C to 121°C**
Sensitivity @100Hz	66 ±4 mV/G
Sensitivity with Excitation	+0.1%/V Typ
Offset with Excitation	-0.5mV/V Typ
Frequency Response	±5%
Mounted Resonance Freq	5500 Hz
Non-Linearity & Hysteresis	±0.20% FSO Typ, ±0.5% FSO Max
Transverse Sensitivity	**2% Max**
Zero Measurand Output	±50 mV Max
Dampening Ratio	0.7
Dampening Ratio Change	+0.08%/°C (-55°C to 121°C)
Thermal Zero Shift	±1.0% FSO Max (0-50°C)
	±2.0% FSO Max (-25-75°C)
Thermal Sensitivity Shift	±2.0% Max (0-50°C)
	±3.0% Max (-25-75°C)
Thermal Transient Error	**<0.001 G/°C**
Threshold (resolution)	**0.008 G's**
Base Strain Sensitivity	**0.01 G's**
Magnetic Susceptibility	**<0.1 G's**
Residual Noise	100uVrms Typ, 500uVrms Max (0.5 to 100 Hz)
	500uVrms Typ, 1mVrms Max (0.5 to 10 KHz)

Gin = G + Gtherm x Trange + Gtrans + Gres + Gbase + Gms

Include the associated errors in the Gin equation and assign each a parameter name. We will now define each of the parameters (including the units will help in determining the equation and avoid any mistakes):

G: input to the accelerometer – the “true” value (G’s)

Gtherm: thermal transient error (G/^{o}C)

Trange: temperature range delta from calibrated temperature (^{o}C)

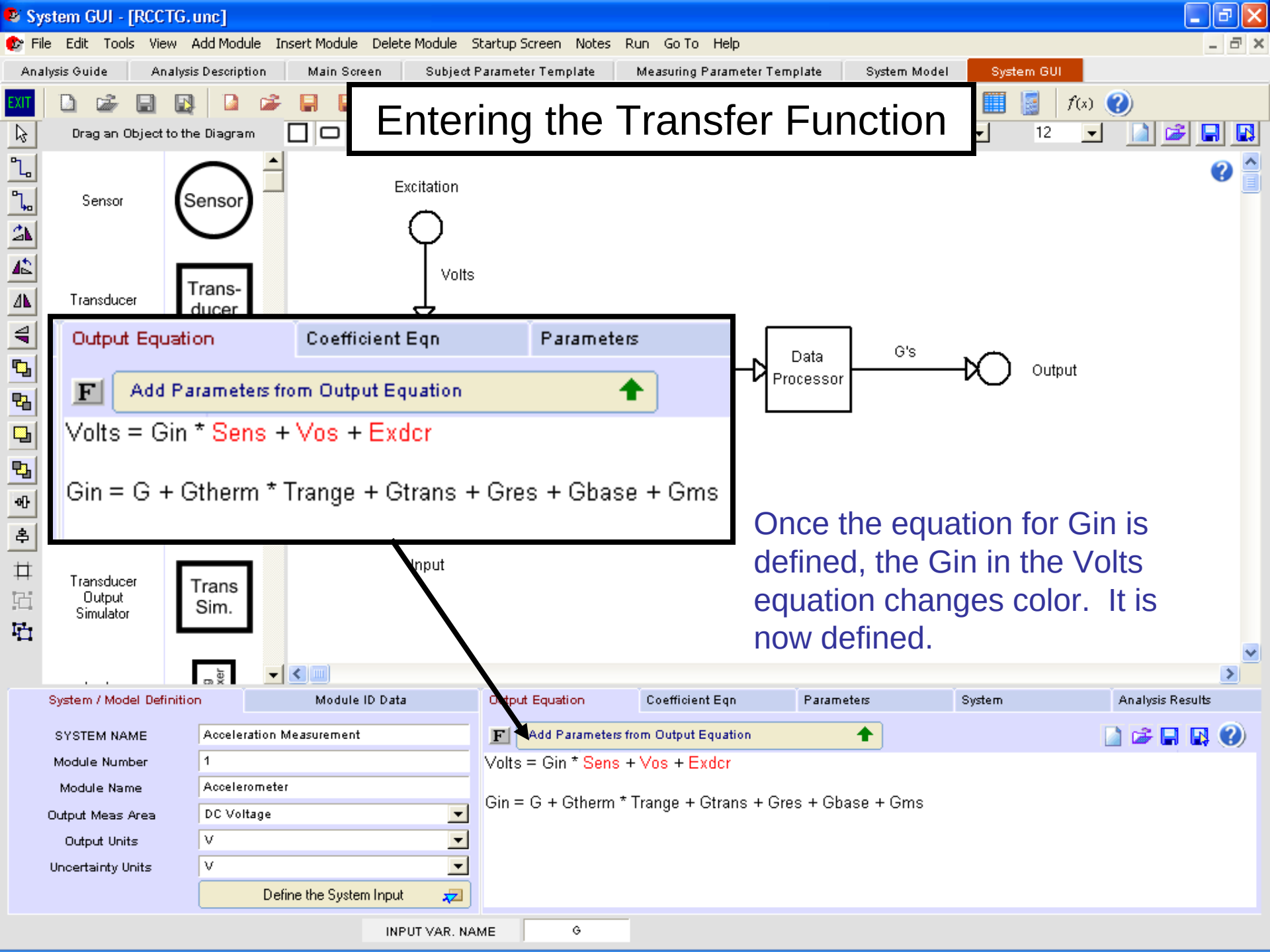
Gtrans: transverse sensitivity (G)

Gres: resolution (G’s)

Gbase: base strain sensitivity (G’s)

Gms: magnetic susceptibility (G’s)

Entering the Transfer Function



Once the equation for Gin is defined, the Gin in the Volts equation changes color. It is now defined.

System GUI - [RCCTG.unc]

File Edit Tools View Add Module Insert Module Delete Module Startup Screen Notes Run Go To Help

Analysis Guide | Analysis Description | Main Screen | Subject Parameter Template | Measuring Parameter Template | System Model | System GUI

Drag an Object to the Diagram

Sensor — Transducer — Power Supply — Reference Standard — Transducer Output Simulator

Excitation → Volts → Sensor → Volts → SCD-108S → Counts → Data Processor → G's → Output

Sensor → G's

INPUT VAR. NAME | G

The input to the entire system is defined to be the variable "G" - the actual acceleration being induced on the accelerometer.

System / Model Definition | Module ID Data | Output Equation | Coefficient Eqn | Parameters | System | Analysis Results

SYSTEM NAME	Acceleration Measurement
Module Number	1
Module Name	Accelerometer
Output Meas Area	DC Voltage
Output Units	V
Uncertainty Units	V

Define the System Input

Add Parameters from Output Equation

Volts = Gin * Sens + Vos + Exdcr

Gin = G + Gtherm * Trange + Gtrans + Gres + Gbase + Gms

INPUT VAR. NAME | G

Gin = G + Gtherm x Trange + Gtrans + Gres + Gbase + Gms

Now each of the terms in the Gin equation must be quantified by determining its nominal value and uncertainty bound. If the bound is something other than 95% and the distribution not normal, it will be noted here. These may be assumptions that are not apparent in the specification sheet.

Starting with the first parameter:

G: input to the accelerometer (G's) – for this analysis, we will make calculations on a input acceleration of 12 G. If you pick one of the calibrated points, it helps in troubleshooting any mistakes you may have in your analysis.

Because this is the input to the measurement system, this information is entered under the "Define System Input"

System GUI - [RCCTG.unc]

File Edit Tools View Add Module Insert Module Delete Module Startup Screen Notes Run Go To Help

Analysis Guide | Analysis Description | Main Screen | Subject Parameter Template | Measuring Parameter Template | System Model | System GUI

Drag an Object to the Diagram

Sensor — Transducer — Power Supply — Reference Standard — Transducer Output Simulator

Excitation
Volts
Sensor
Volts
SCD-108S
Counts
Data Processor
G's
Output
G's

Define the System Input

To define the input, click on "Define the System Input"

System / Model Definition | Module ID Data | Output Equation | Coefficient Eqn | Parameters | System | Analysis Results

SYSTEM NAME	Acceleration Measurement
Module Number	1
Module Name	Accelerometer
Output Meas Area	DC Voltage
Output Units	V
Uncertainty Units	V

Define the System Input

Add Parameters from Output Equation

Volts = Gin * Sens + Vos + Exdcr

Gin = G + Gtherm * Trange + Gtrans + Gres + Gbase + Gms

INPUT VAR. NAME G

Define System Input

Subject Parameter Bias Uncertainty Worksheet - [RCCTG.unc]

OK Cancel Procedure Edit Get Item Reliability

GENERAL PROCEDURE

Toleranced Parameter

Reference Standard

Containment Limits

Enter identifying information, e.g., **Manufacturer**, **Model**, **Serial Number** and **Parameter Name**.

Select the Measurement Area, enter the **Nominal** or **Reading** value, select the Nominal Units, and the Tolerance Units, and enter **Range** and **Full Scale** specs (if applicable).

Specification Options

Select the basis for computing or entering the

Measurement Area	Acceleration
Nominal Units	g
Tolerance Units	g
Nominal Value	12 g
Upper Range Limit	30 g
Lower Range Limit	-30 g
Full Scale	60 g

Parameter Data

Description	Item Data
Manufacturer	Endevco
Model Number	4790A-30
Option	
Description	Variable Capacitance Accelerometer
Class Description	
Class Code	
Serial Number	AHGM8

Specification Options

Toleranced Parameter Reference Standard Containment Limits

Display Units

Nominal Tolerance Units

Symmetric Single-Sided Upper Single-Sided Lower

Other (Define) Spec Master

% of Full Scale ppm of Full Scale % of Range ppm of Range Subject Parameter ± Tolerance Limits % OOT

Summary Data

Measurement Area	Acceleration
Nominal Units	g
Tolerance Units	g
Nominal Value	12 g
Upper Range Limit	30 g
Lower Range Limit	-30 g
Full Scale	60 g

Item Data Parameter Data

Description	Item Data
Manufacturer	Endevco
Model Number	4790A-30
Option	
Description	Variable Capacitance Accelerometer
Class Description	
Class Code	
Serial Number	AHGM8

Enter the information for the accelerometer along with the measurement range and units. Click "OK" when finished.

Gin = G + Gtherm x Trange + Gtrans + Gres + Gbase + Gms

The next parameter varies with temperature, so we will have to define the temperature range of the environment where the measurement will be made.

Gtherm: thermal transient error (G/ºC)

Nominally it should be at 0 G /ºC and has a 2σ value of 0.001 G/ºC.

Trange: temperature range delta from calibrated temperature (ºC)

From the transducer lab calibration report, the calibration was performed at a temperature of 22.84ºC (73ºF). The expected temperature range the transducer will be from -55ºC to 121ºC. The largest temperature range delta from the calibrated temperature will be 121 - 22.84 = 98.16ºC.

This information will be entered in a different screen called the **Error Source Uncertainty Worksheet** accessed in the **System Model** window.

Accessing the System Model Screen

Click the **System Model** tab.

System GUI - [RCCTG.unc]

File Edit Tools View Add Module Insert Module Delete Module Startup Screen Notes Run Go To Help

Analy... Template | Measuring Parameter Template | System Model | System GUI

Sensor | Transducer | Power Supply | Reference Standard | Transducer Output Simulator

Excitation | Volts | Sensor | Volts | SCD-108S | Counts | Data Processor | G's | Output | G's | Input

System / Model Definition | Module ID Data | Output Equation | Coefficient Eqn | Parameters | System | Analysis Results

SYSTEM NAME: Acceleration Measurement
Module Number: 1
Module Name: Accelerometer
Output Meas Area: DC Voltage
Output Units: V
Uncertainty Units: V

Define the System Input

Add Parameters from Output Equation

Volts = Gin * Sens + Vos + Exdcr

Gin = G + Gtherm * Trange + Gtrans + Gres + Gbase + Gms

Sens = Snom + Snom * Sexc * Vexc + Stherm

Vos = Voscal + Vosexc * Vexc + Vostherm

INPUT VAR. NAME: G

System Model Screen

System Model - [RCCTG.unc]

File Edit View Tools Startup Screen Run Notes

Analysis Guide | Analysis Description | Main Scre | System Model | System GUI

Procedure

- Define the System Input
- Identify the System and
- Set th Area
- Enter the Module Output Equation
- Define the Module Parameters
- Establish the Correlations
- Update the Notes
- Select Report Options
- Print the Module Report
- Update the Module Library

System / Module Definition | Module ID Data

SYSTEM NAME Acceleration Measurement

Module Name Accelerometer

Module Output Equation

Volts = Gin * Sens + Vos + Exdcr

Gin = G + Gtherm * Trange + Gtrans + Gres + Gbase + Gms

Module Equations from System GUI carry over to the System Model screen

Module Parameters

Add Parameters from Output Equation

Edit	Description	Parameter Name	Standard Uncertainty	% Confidence	± Error Limits	Coefficient	Component Uncertainty (V)
	Module Input Variable	G	0.000 g	-	-	0.066	0.000 V

Insert Module | Delete Module | Add Module | Total Module Uncertainty | V

To begin defining each parameter, click on the "Add Parameters from Output Equation" button.

SYSTEM ANALYSIS

Module Name	Input	Output	Uncertainty	Coefficient
Accelerometer	12 g	0.0 V	V	0
SCD-108S	0.0 V	2,849.7 Counts	41.6 Counts	0
Processor	2,849.7 Counts	12.0 g	0.658 g	0

Series System | Update

System Input	12 g
System Output	12.0 g
Total System Uncertainty	g
Confidence Level (%)	95.00 %
Coverage Factor	1.9600
Degrees of Freedom	
Tolerance Limits	0.00 g

System Model - [RCCTG.unc]

File Edit View Tools Startup Screen Run Notes Go To Help

Analysis Guide | Analysis Description | Main Screen | Subject Parameter Template | Measuring Parameter Template | System Model | System GUI

Procedure

- ✓ Define the System Input
- ✓ Identify the System and Module
- ✓ Set the Measurement Area and Units
- ✓ Enter the Module Output Equation
- ✓ Define the Module Parameters
- ✗ Establish the Correlations
- ✗ Update the Notes
- ✗ Select Report Options
- ✗ Print the Module Report
- ✗ Update the Module Library

System / Module Definition | Module ID Data

SYSTEM NAME	Acceleration Measurement
Module Name	Accelerometer
Module Number	1
Output Meas. Area	DC Voltage
Output Units	V
Uncertainty Units	V

Module Output Equation

Volts = Gin * Sens + Vos + Exdcr

Gin = G + Gtherm * Trange + Gtrans + Gres + Gbase + Gms

The parameters now show up in the table below.

Module Parameters

Edit	Description	Parameter Name	Standard Uncertainty	% Confidence	Component Uncertainty (V)
		Sens	0		0
		Vos	0		0
		Exdcr	0		0
		Gtherm			
		Trange			
		Gtrans			
		Gres			
	Module Input Variable	G	0.000 g	-	0.000 V

Insert Module | Delete Module | Add Module

SYSTEM ANALYSIS

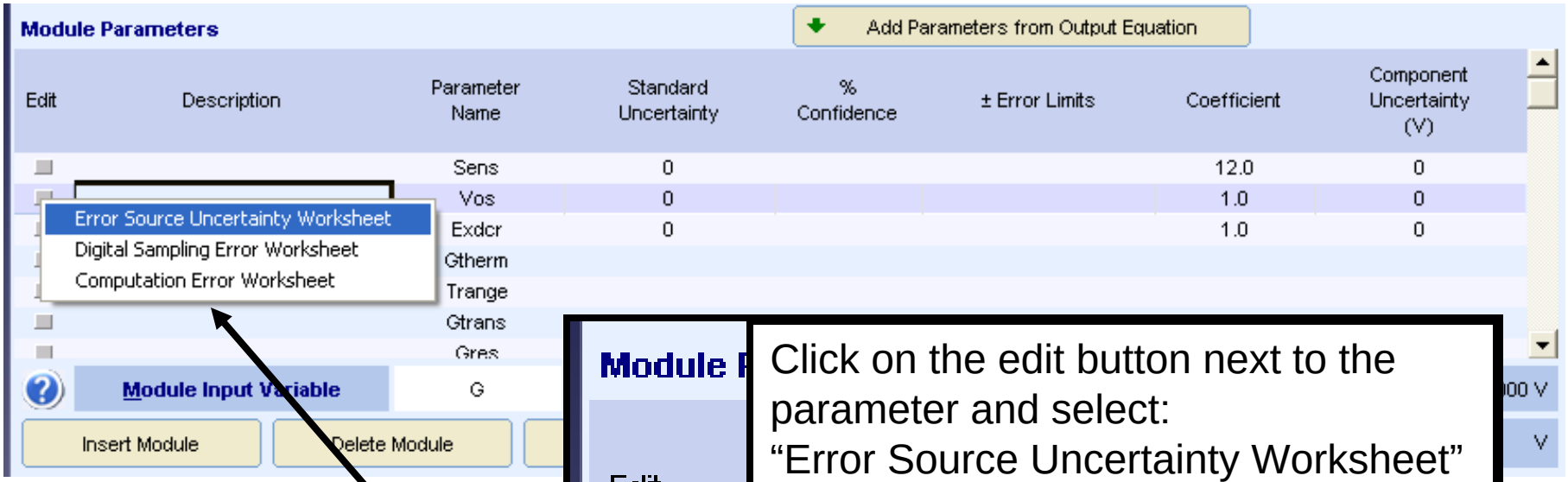
Module Name	Input	Output	Uncertainty
Accelerometer	12 g	0.0 V	V
SCD-108S	0.0 V	2,849.7 Counts	41.6 Counts
Processor	2,849.7 Counts	12.0 g	0.658 g

Update

	12 g
	12.0 g
	g
Confidence Level (%)	95.00 %
Coverage Factor	1.9600
Degrees of Freedom	
Tolerance Limits	0.00 g

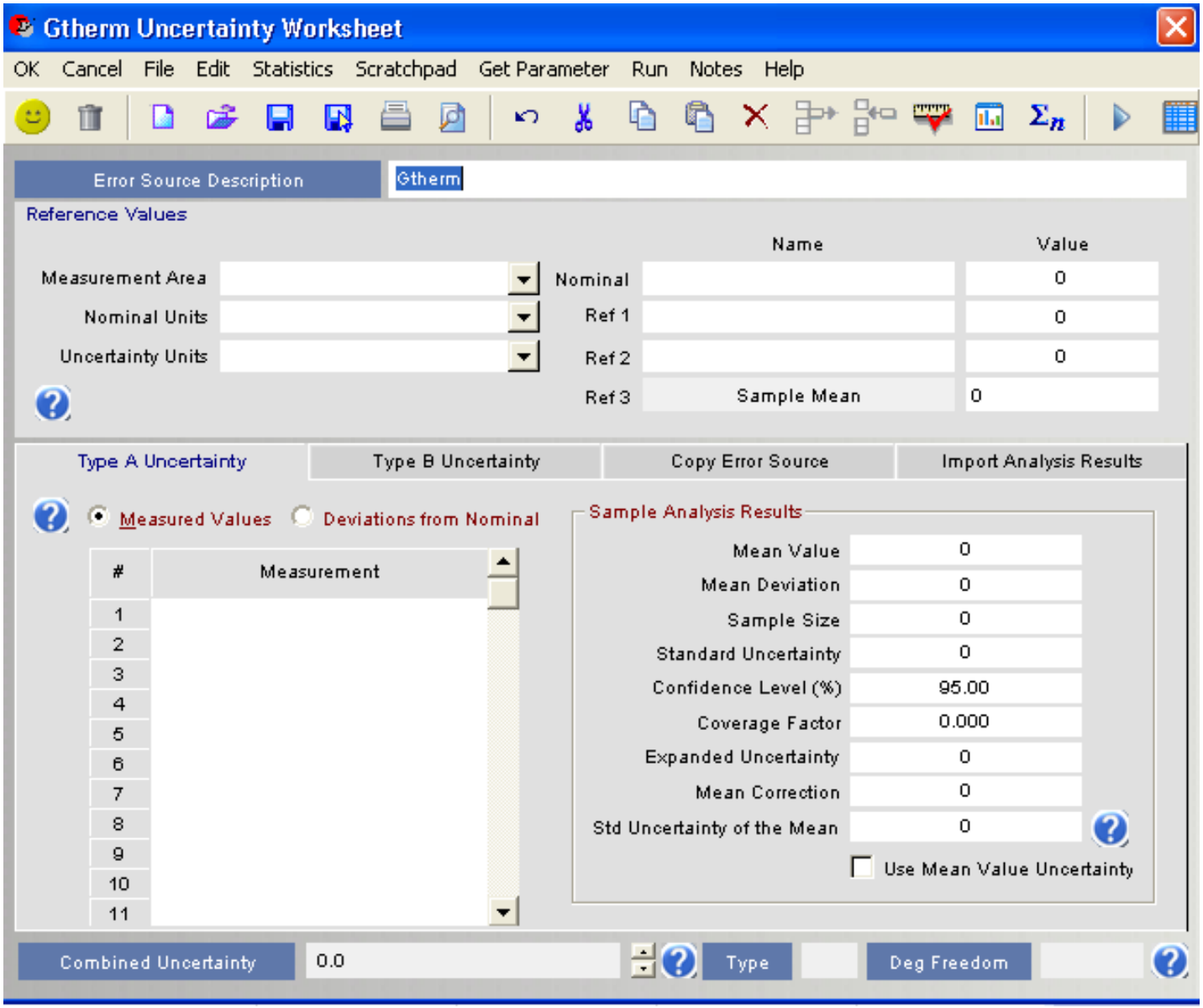
Parameter Name
Sens
Vos
Exdcr
Gtherm
Trange
Gtrans
Gres

Module Parameters Table | Row 1



Click on the edit button next to the parameter and select:
"Error Source Uncertainty Worksheet"

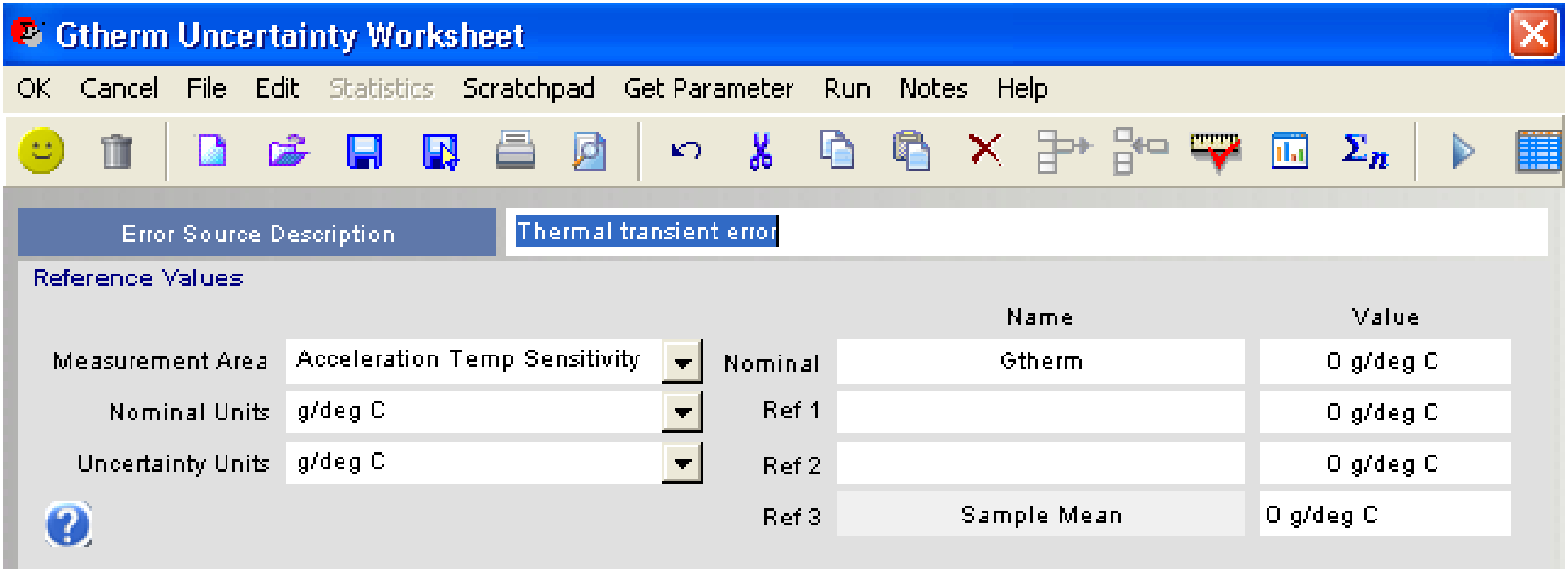
The Gtherm Uncertainty Worksheet opens.



Gtherm: thermal transient error (G/ºC)

Nominally it should be at 0 G /ºC and has a 2σ value of 0.001 G/ºC.

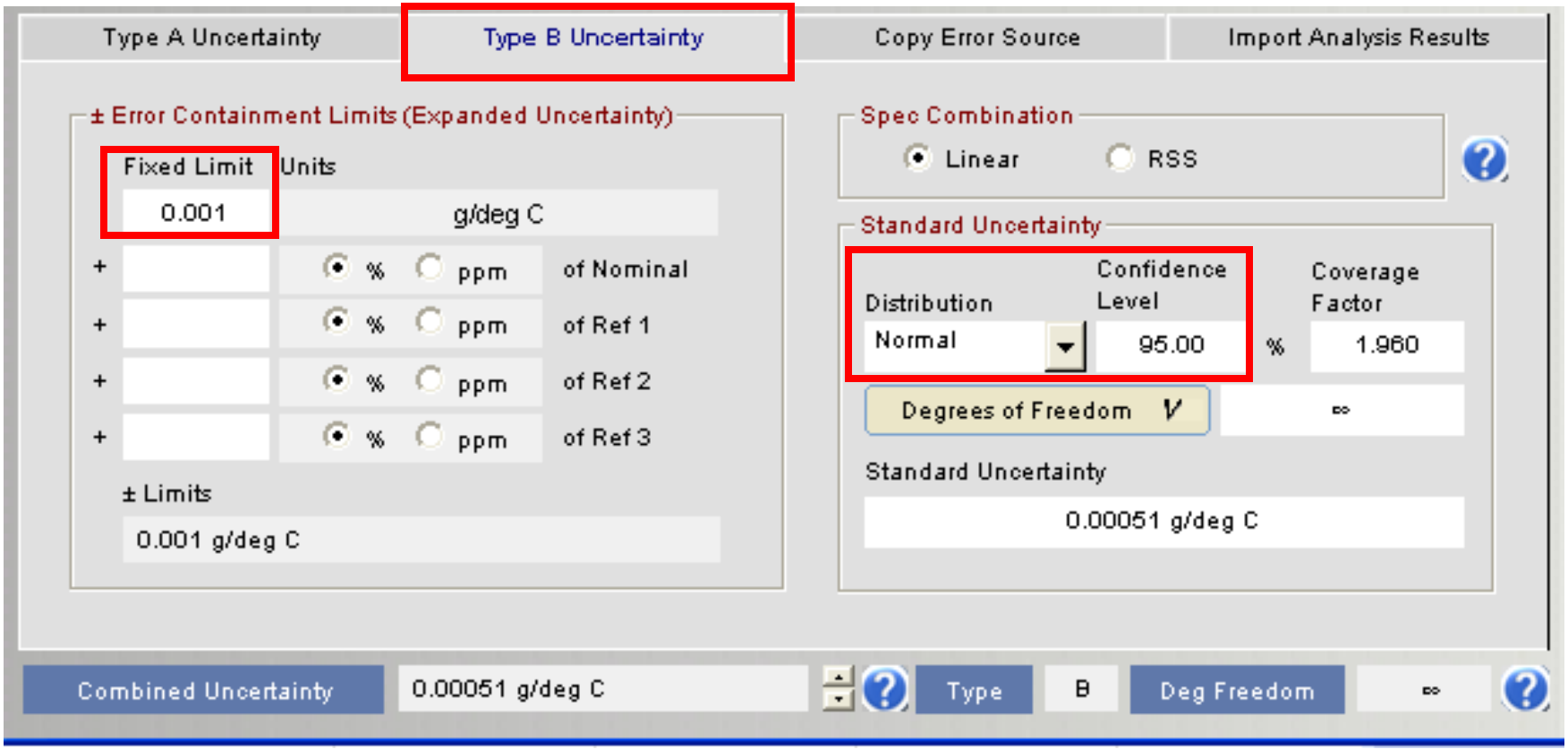
Enter the name of the parameter, the measurement area, the nominal and uncertainty units, and the nominal value.

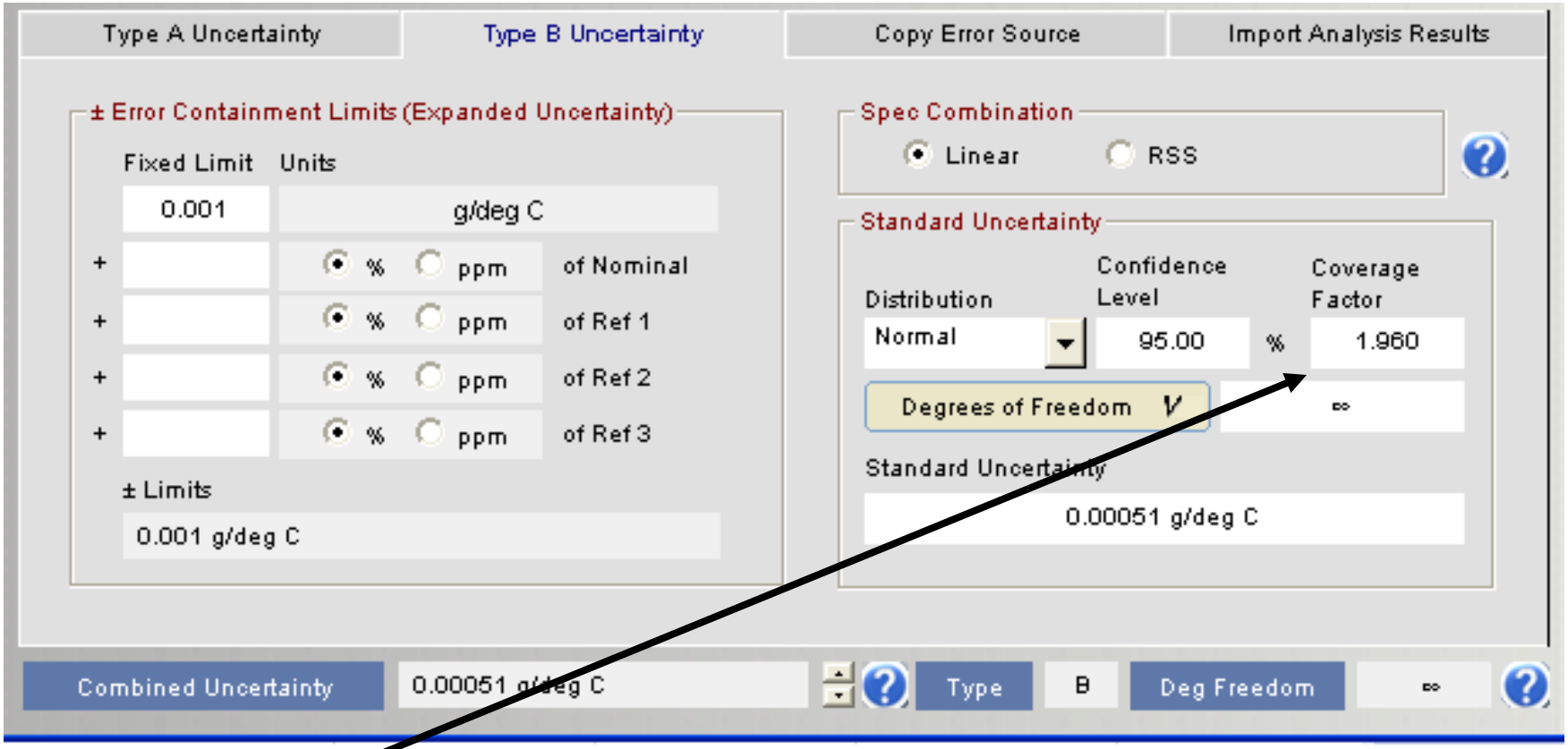


Gtherm: thermal transient error (G/ºC)

Nominally it should be at 0 G /ºC and has a 2σ value of 0.001 G/ºC.

Because we are working off the specification sheet of the accelerometer, we will be doing a "Type B" uncertainty. Enter the assumed 2σ value for the fixed limit and select the normal distribution and corresponding confidence level of 95%.





Coverage Factor

1.960

Note that the coverage factor (in number of standard deviations) is not 2. This is the <u>exact</u> number of σ's for a 95.0% Confidence Level.

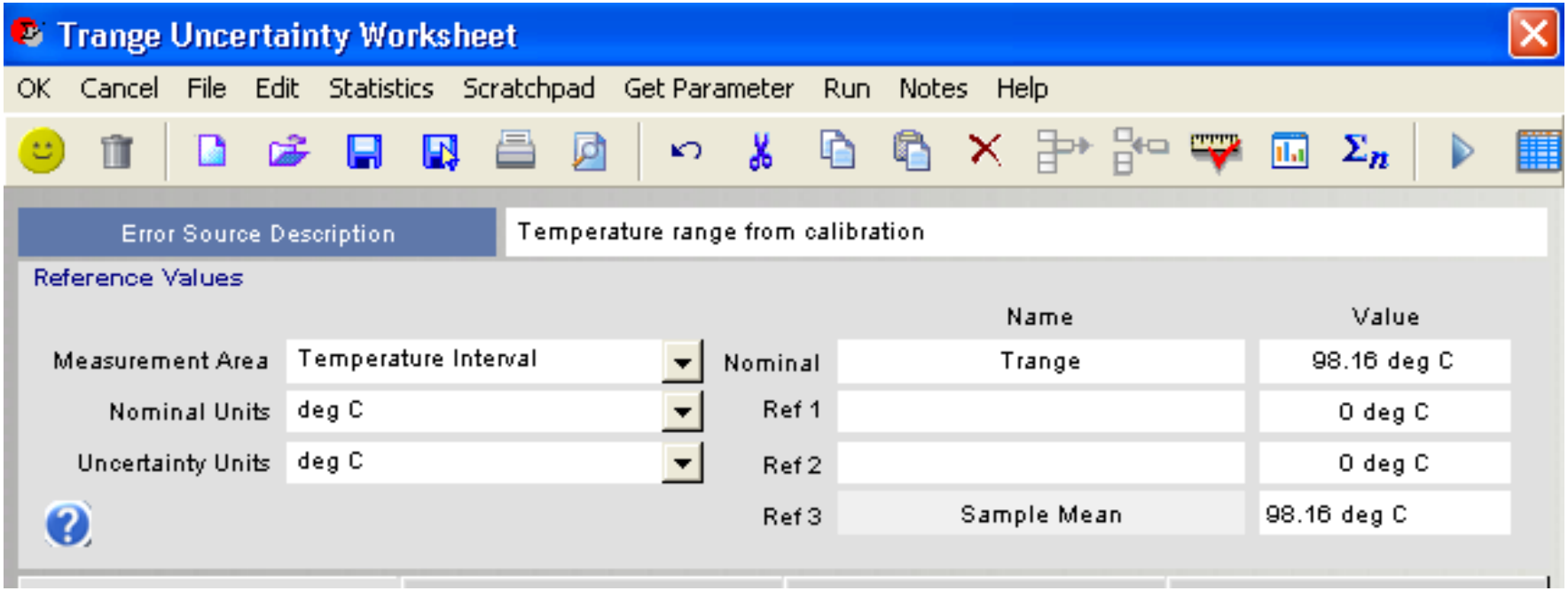
Click OK or the smiley to return to the System Model screen.

Similarly, for the temperature range, Trange:

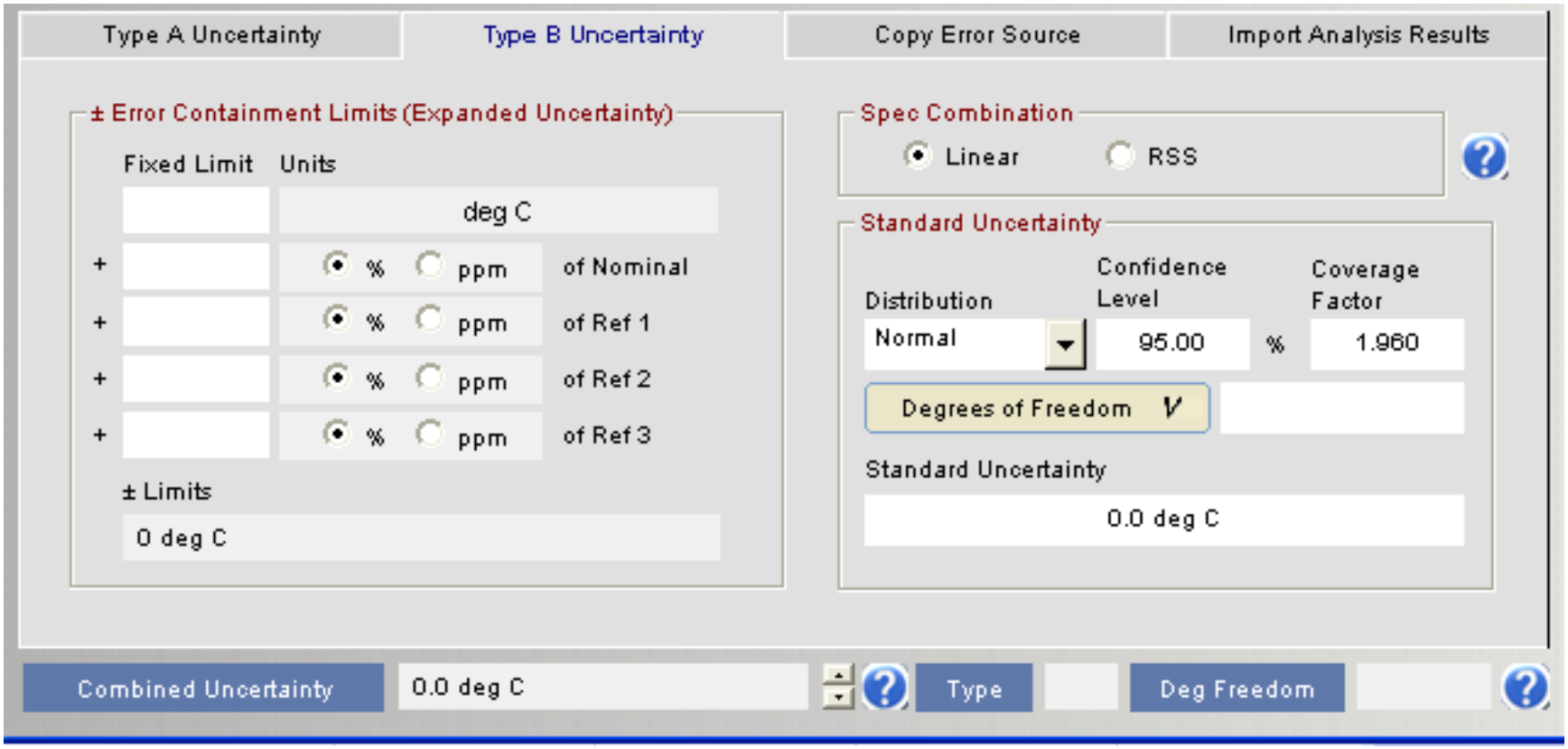
Trange: temperature range delta from calibrated temperature (ºC)

From the transducer lab calibration report, the calibration was performed at 22.84ºC. The expected temperature range the transducer will see is from -55ºC to 121ºC. The largest temperature range delta from the calibrated temperature will be 121 - 22.84 = 98.16ºC.

We are assuming the worst case in temperature deviation from the calibrated temperature.



Here is where a stated assumption would occur. I am going to assume the temperature being read in the calibration lab has no uncertainty. So no information will be entered for the “Type B Uncertainty”.



Click OK or the smiley to return to the System Model screen.

Gin = G + Gtherm x Trange + Gtrans + Gres + Gbase + Gms

This parameter will be entered in differently from the previous entries because it is dependent upon the maximum magnitude of the accelerations in the other two axes.

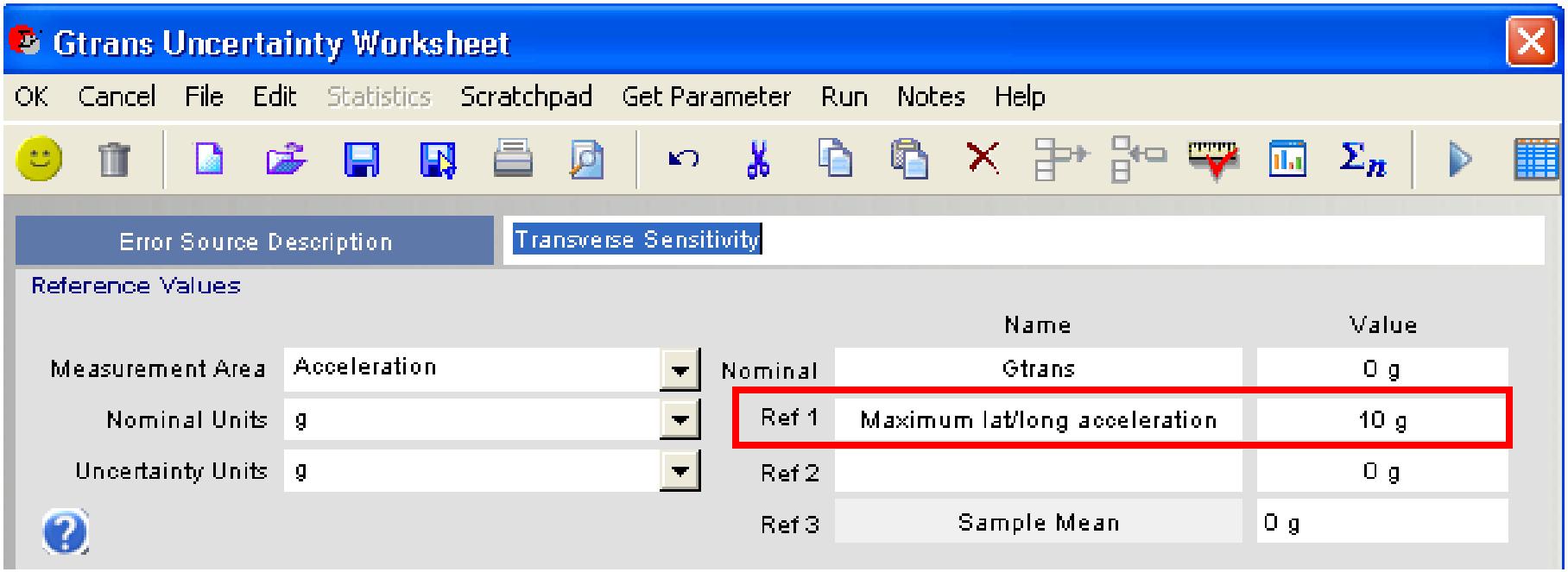
Gtrans: transverse sensitivity (G)

This is how much of the lateral and longitudinal acceleration is sensed by the accelerometer and contributes to the normal acceleration. The specifications state that it is 2% of the other axes. The maximum lateral or longitudinal acceleration for this test is expected to be 10 G's.

The G levels in the other axes (if not included in your list of parameters) will have to be defined by the flight test engineer. They are writing the test plans and should know these quantities.

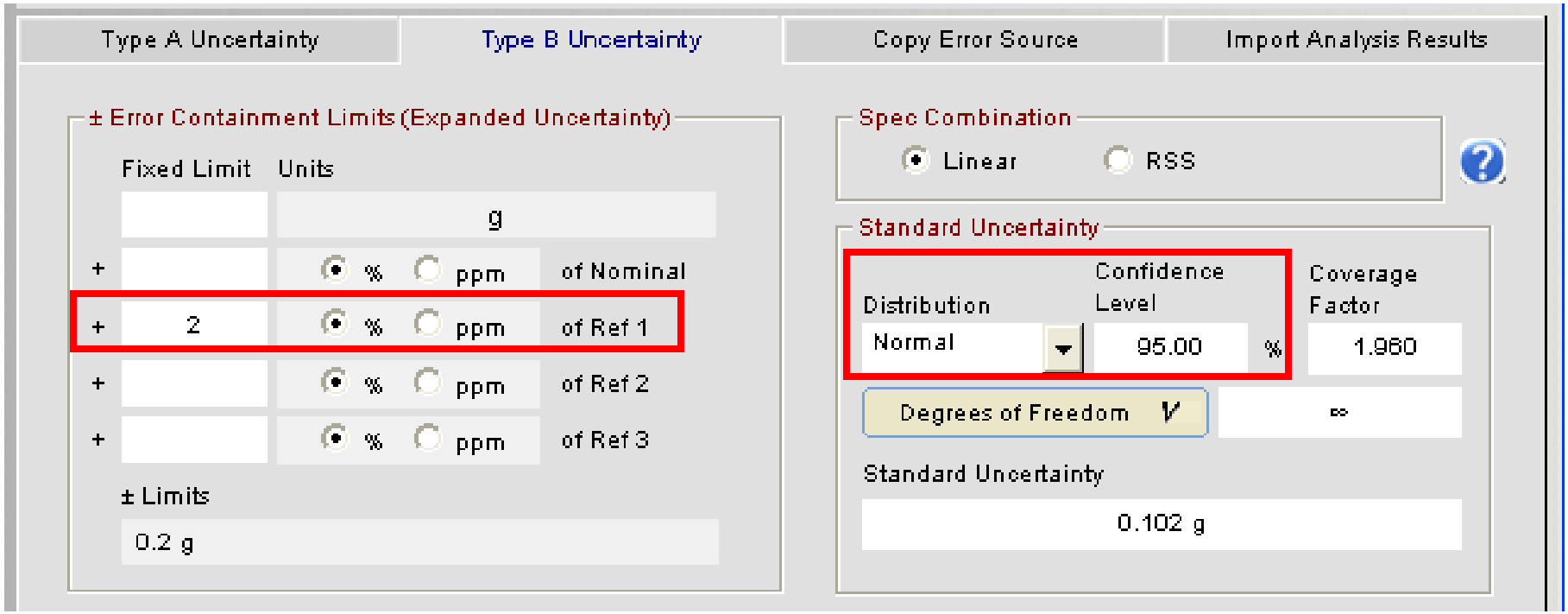
The specifications state that it is 2% of the other axes. The maximum lateral or longitudinal acceleration for this test is expected to be 10 G's.

To enter the maximum G level of the other axes, use the reference (Ref 1). This will be the expected maximum lateral or longitudinal acceleration of 10 G.



The specifications state that it is 2% of the other axes. The maximum lateral or longitudinal acceleration for this test is expected to be 10 G's.

Under the Type B Uncertainty, we enter that the error is 2% of Ref 1, which is our maximum lateral or longitudinal acceleration. A normal distribution is assumed with a confidence level of 95%.

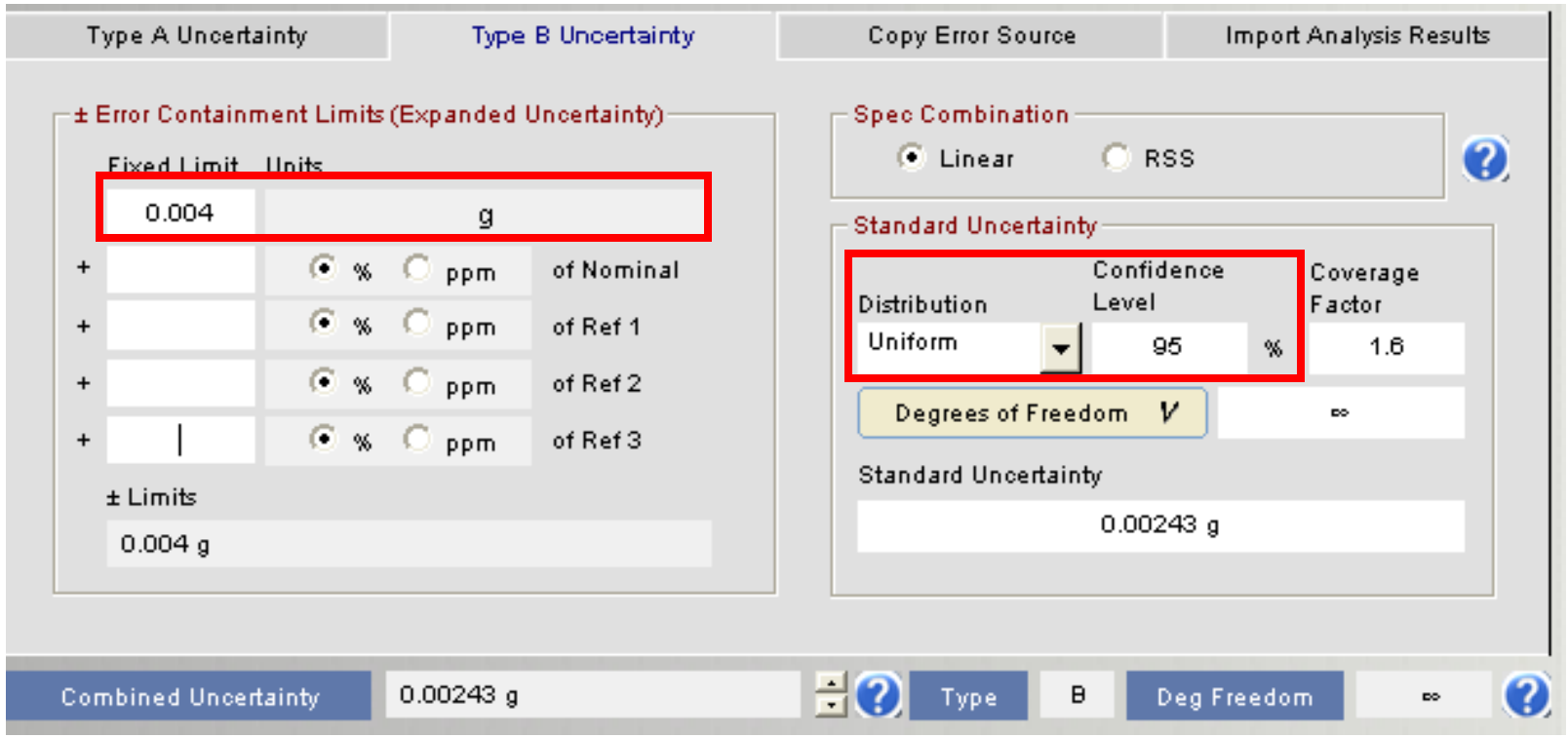


Gin = G + Gtherm x Trange + Gtrans + Gres + Gbase + Gms

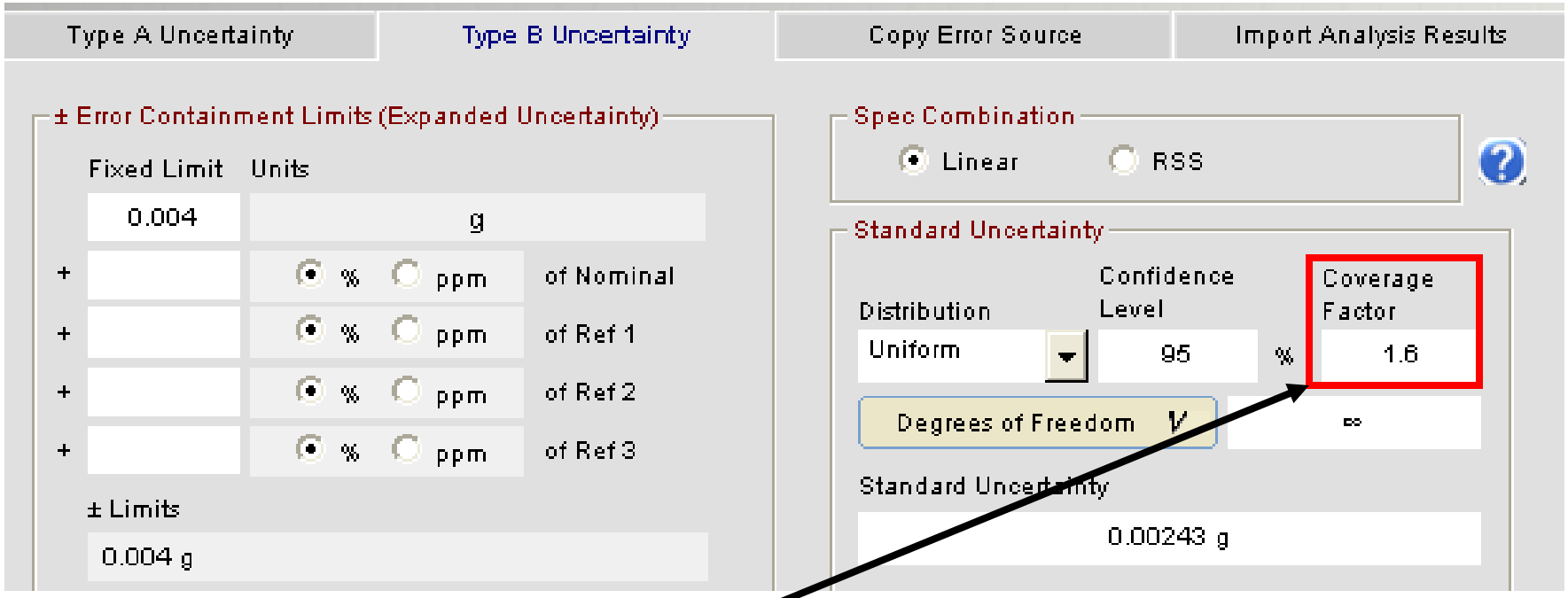
Gres: resolution (G's)

The resolution of the accelerometer is reported to be 0.008G. The error would be half of that resolution. Resolution is usually assumed to be a *uniform distribution* (an equal probability of being above or below the threshold).

½(0.008G) = 0.004 G



Coverage Factor for the Uniform Distribution



With a 95% Confidence Level, the coverage factor for a Uniform Distribution is 1.6σ, not 1.96σ which is for a Normal Distribution.

Gin = G + Gtherm x Trange + Gtrans + Gres + Gbase + Gms

The remaining two parameters will be similarly entered in the Uncertainty Worksheet.

Gbase: base strain sensitivity (G's)

Assumed to be 0 nominally, and varies by 0.01 G.

Gms: magnetic susceptibility (G's)

Is nominally 0 and is maximally 0.1 G

Volts = Gin x Sens + Vos + Exdcr

Sens is the voltage sensitivity of the accelerometer to the input. Again, read through the specification sheet and determine which errors will contribute to the sensing of the acceleration.

Range	±30 Gpk
Full Scale Output (FSO)	4 volts
Operating Temp Range	-55°C to 121°C
Sensitivity @100Hz	66 ±4 mV/G
Sensitivity with Excitation	**+0.1%/V Typ**
Offset with Excitation	-0.5mV/V Typ
Frequency Response	±5%
Mounted Resonance Freq	5500 Hz
Non-Linearity & Hysteresis	±0.20% FSO Typ, ±0.5% FSO Max
Transverse Sensitivity	2% Max
Zero Measurand Output	±50 mV Max
Dampening Ratio	0.7
Dampening Ratio Change	+0.08%/°C (-55°C to 121°C)
Thermal Zero Shift	±1.0% FSO Max (0-50°C)
	±2.0% FSO Max (-25-75°C)
Thermal Sensitivity Shift	**±2.0% Max (0-50°C)**
	±3.0% Max (-25-75°C)
Thermal Transient Error	<0.001 G/°C
Threshold (resolution)	0.008 G's
Base Strain Sensitivity	0.01 G's
Magnetic Susceptibility	<0.1 G's
Residual Noise	100uVrms Typ, 500uVrms Max (0.5 to 100 Hz)
	500uVrms Typ, 1mVrms Max (0.5 to 10 KHz)

Sens = Snom + Sexc x Vexc + Stherm

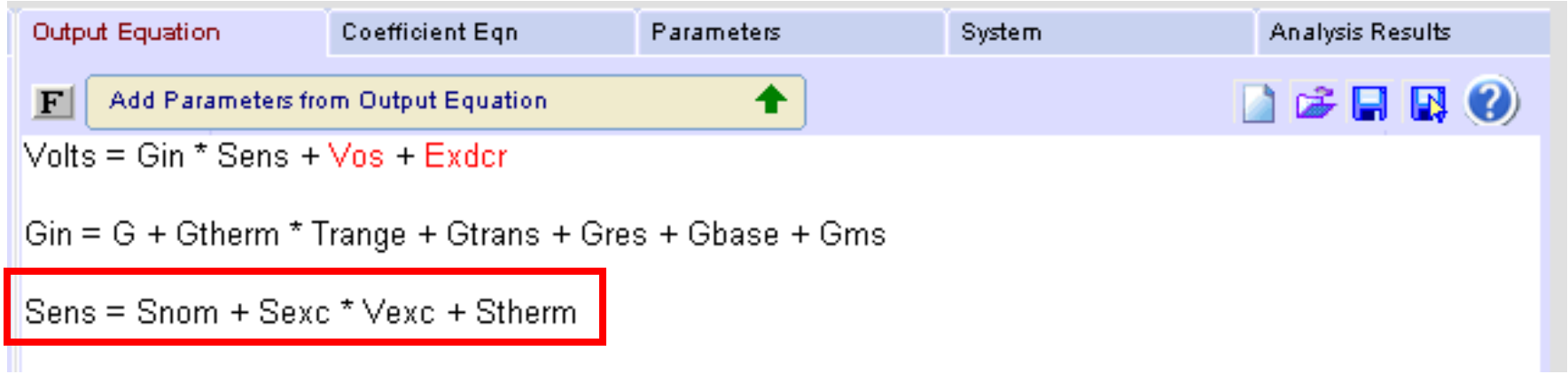
Now each of the terms in the Sens equation must be defined and entered into the Output Equation list in the System GUI:

Snom: the nominal sensitivity from the laboratory calibration (V/G's)

Sexc: sensitivity error due to the excitation voltage (%/V)

Vexc: the excitation applied to the accelerometer (V)

Stherm: thermal sensitivity shift (V/G)



Going back to the System Model screen, the parameters for the new equation can be added to the Module Parameters list by clicking the “Add Parameters from Output Equation”.

Follow the same procedure as before to enter in the errors in the Uncertainty Worksheet for each parameter.

Note any assumptions being made as the worksheet is filled in.

Module Parameters → Add Parameters from Output Equation

Edit	Description	Parameter Name	Standard Uncertainty	% Confidence	± Error Limits	Coefficient	Component Uncertainty (V)
	Base Strain	Gbase	0.0051 g	95.00	0.0100	0.0660	0.000337
	Magnetic Susceptiblility	Gms	0.051 g	95.0	0.100	0.0660	0.00337
	Nominal Sensitivity	Snom	0.0001132 V/g	95.00	0.0002218	12.0	0.0013580
	Voltage Excitation Sensitivity	Sexc	0.000033695 V/V/g	95.00	0.000066042	7.92	0.000267035
	Voltage excitation	Vexc	0.0153 V	95.00	0.0300		0.0000
	Thermal Sensitivity	Stherm	0.00101086 V/g	95.00	0.00198125	12.0	0.01213032
	Module Input Variable	G	0.000 g	-	-	0.066	0.000 V

Insert Module | Delete Module | Add Module | **Total Module Uncertainty** 0.0434 V

Sens = Snom + Sexc x Vexc + Stherm

Snom: the nominal sensitivity from the laboratory calibration (V/G's)

The nominal sensitivity is obtained from the transducer laboratory calibration report. Over the 0 to 300 Hz frequency range, the average sensitivity was 66.0416 mV/G with a standard deviation of 0.1109 mV/G. Be sure to change the units from mV/G to V/G so the units match.

The nominal sensitivity is 0.0660416 V/G. To obtain the 95% bound, the standard deviation is multiplied by 2 (which would be 2σ).
2(0.0001109 V/G) = 0.0002218 V/G

Sens = Snom + Sexc x Vexc + Stherm

Sexc: sensitivity error due to the excitation voltage ((V/G)/V)

Nominally Sexc will be 0. The spec says this varies by 0.1% of Snom per volt of excitation. We know the nominal sensitivity, Snom from the transducer laboratory calibration.

Vex: the excitation applied to the accelerometer (V)

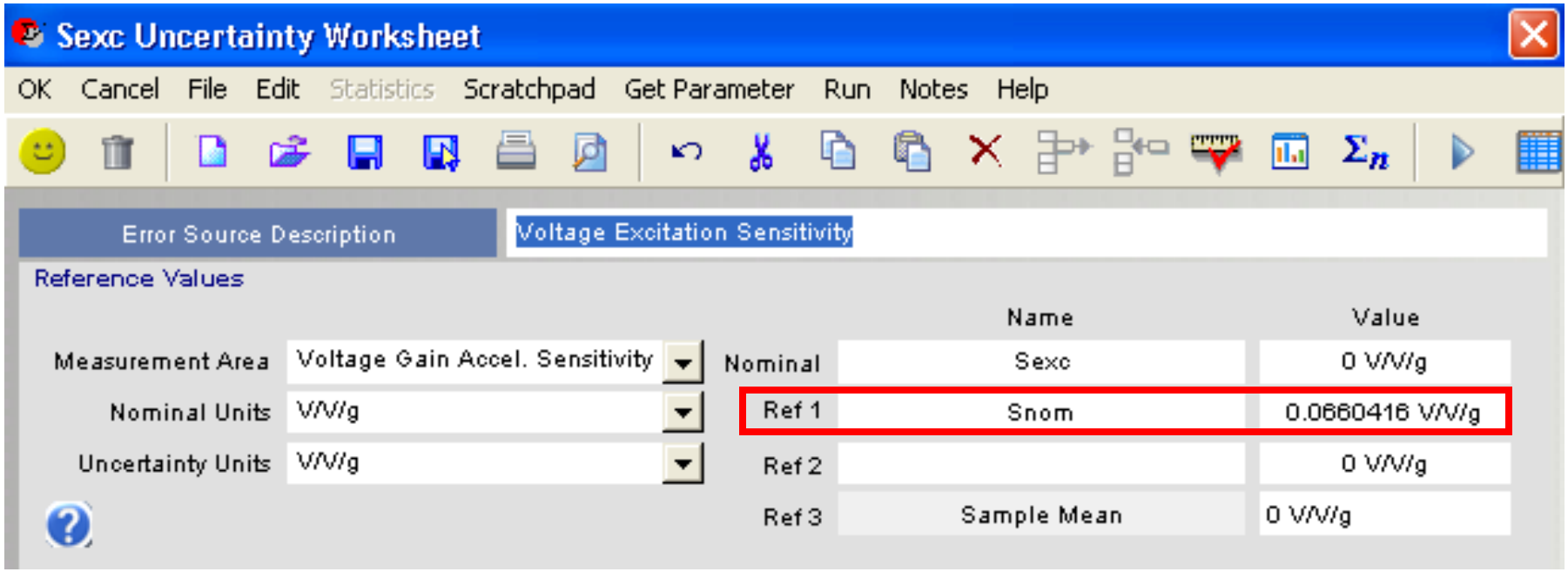
The excitation voltage used will have a nominal value of 10V. From the TTC specification of the SCD-108S, the voltage can vary by 0.3%.

The sensitivity error due to the excitation is defined as a percentage of the nominal sensitivity per every volt of excitation. Therefore, this quantity is multiplied by the excitation voltage, which also has an error associated with it.

The entries for Sexc and Vexc in the Uncertainty Worksheet will look like this:

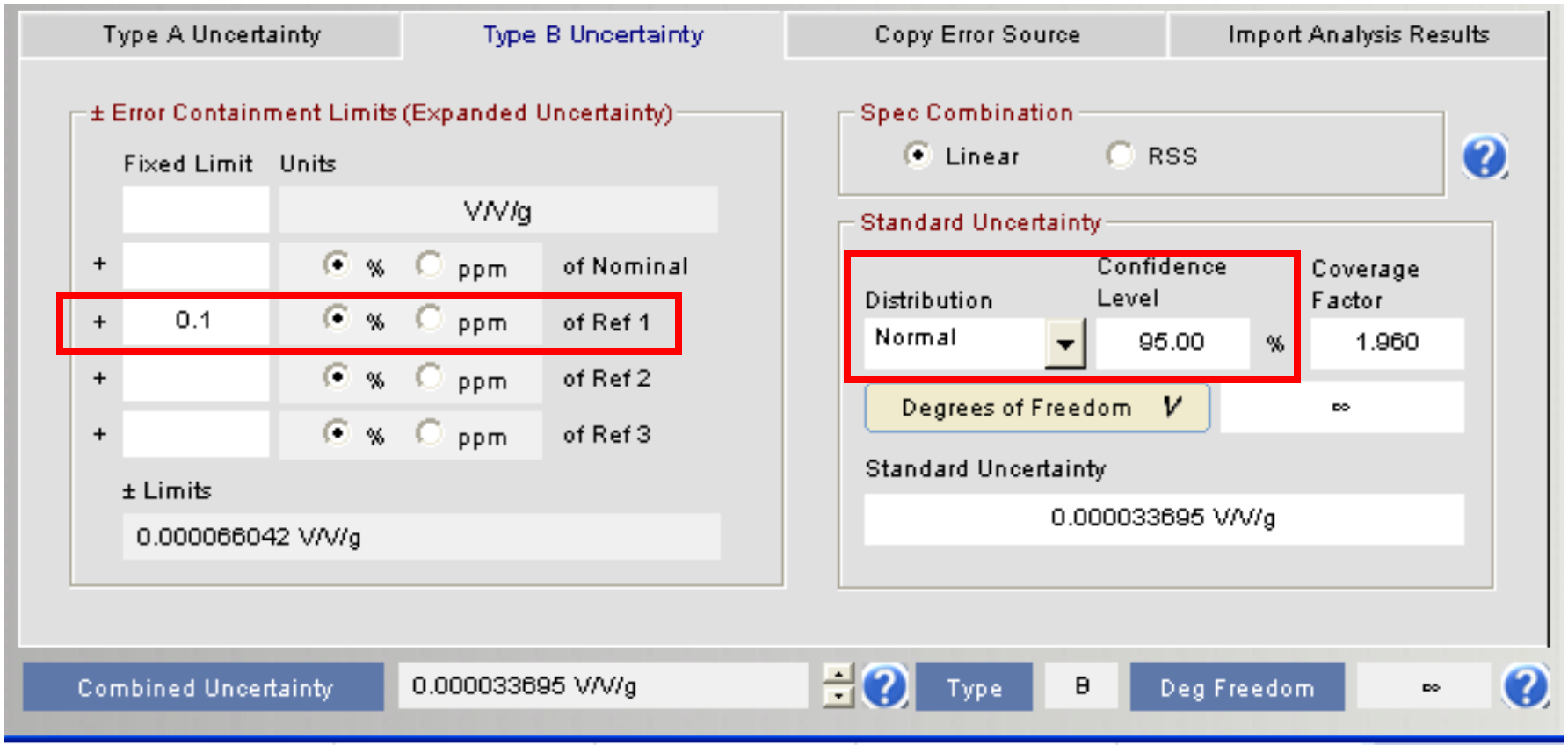
The units chosen for the Voltage Excitation Sensitivity will be V/V/g, or written another way (V/g)/V, because it is an acceleration sensitivity per volt. When multiplied by the excitation voltage, you will get a sensitivity error in V/g. This matches the units we used to define sensitivity.

Because Sexc is defined as a percentage of the nominal sensitivity, we will utilize Ref 1 for the nominal sensitivity value.



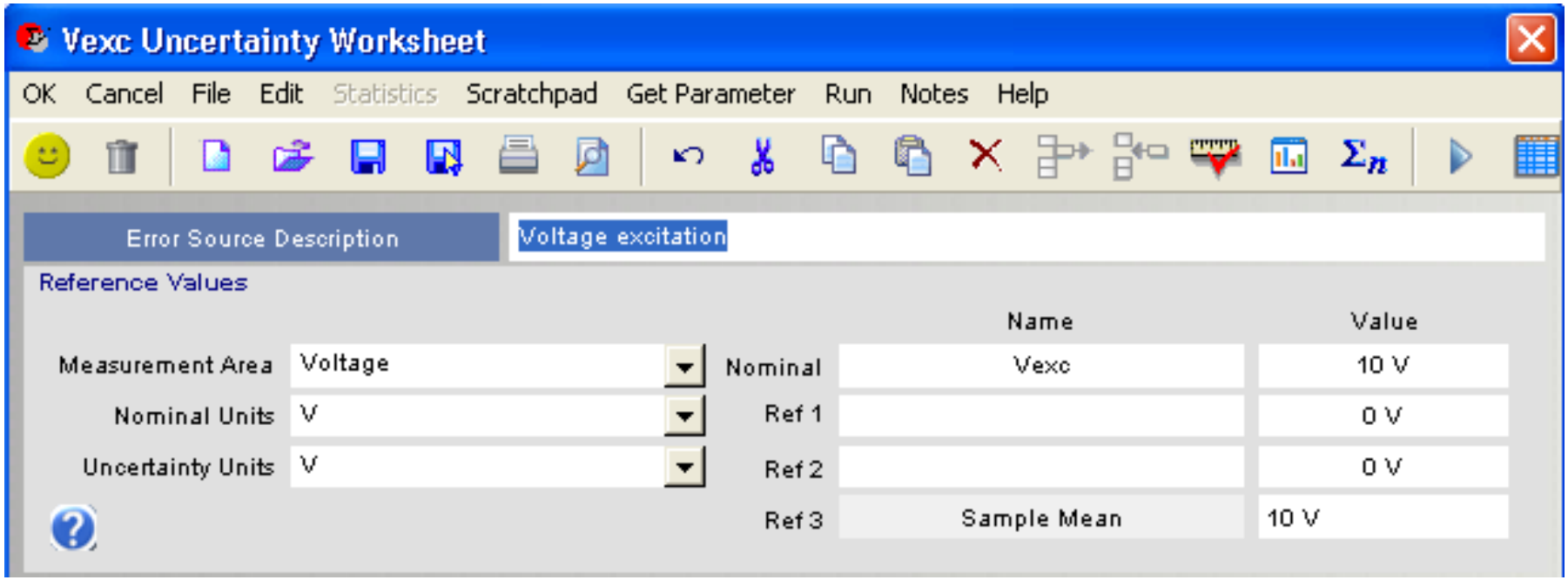
Sexc: sensitivity error due to the excitation voltage ((V/G)/V)

Nominally Sexc will be 0. The spec says this varies by 0.1% of Snom per volt of excitation. We know the nominal sensitivity, Snom from the transducer laboratory calibration.



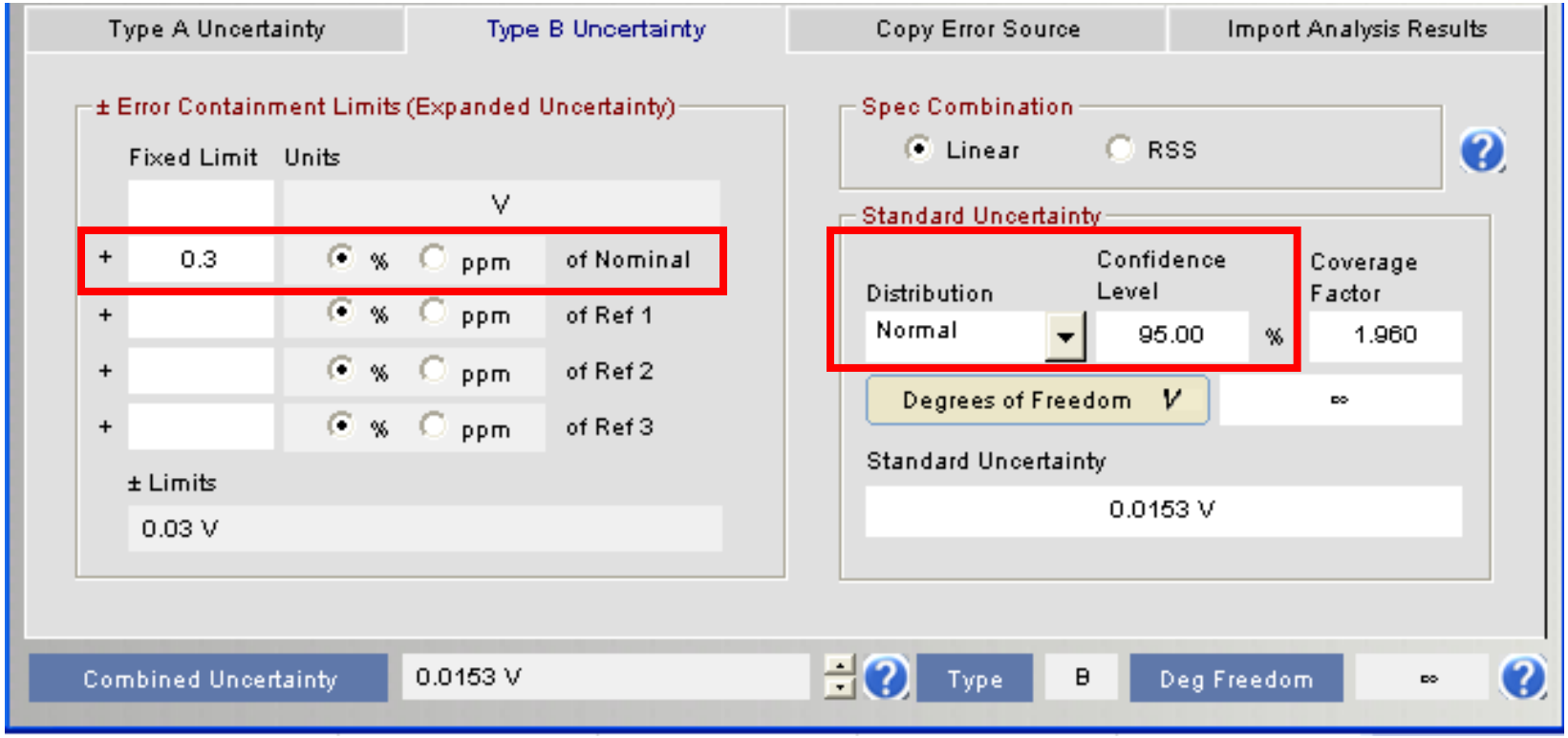
Vex: the excitation applied to the accelerometer (V)

The excitation voltage used will have a nominal value of 10V. From the TTC specification of the SCD-108S, the voltage can vary by 0.3%.



Vex: the excitation applied to the accelerometer (V)

The excitation voltage used will have a nominal value of 10V. From the TTC specification of the SCD-108S, the voltage can vary by 0.3%.



Sens = Snom + Sexc x Vexc + Stherm

Stherm: thermal sensitivity shift (V/G)

For the temperature range of -25 to 75ºC, the error is 3% of the nominal sensitivity.

Volts = Gin x Sens + Vos + Exdcr

Vos is the output of the accelerometer at a 0 G input. Looking through the specifications, errors which contribute to the offset voltage will be considered.

Range	±30 Gpk
Full Scale Output (FSO)	4 volts
Operating Temp Range	-55°C to 121°C
Sensitivity @100Hz	66 ±4 mV/G
Sensitivity with Excitation	+0.1%/V Typ
Offset with Excitation	**-0.5mV/V Typ**
Frequency Response	±5%
Mounted Resonance Freq	5500 Hz
Non-Linearity & Hysteresis	±0.20% FSO Typ, ±0.5% FSO Max
Transverse Sensitivity	2% Max
Zero Measurand Output	±50 mV Max
Dampening Ratio	0.7
Dampening Ratio Change	+0.08%/°C (-55°C to 121°C)
Thermal Zero Shift	**±1.0% FSO Max (0-50°C)**
	±2.0% FSO Max (-25-75°C)
Thermal Sensitivity Shift	±2.0% Max (0-50°C)
	±3.0% Max (-25-75°C)
Thermal Transient Error	<0.001 G/°C
Threshold (resolution)	0.008 G's
Base Strain Sensitivity	0.01 G's
Magnetic Susceptibility	<0.1 G's
Residual Noise	100uVrms Typ, 500uVrms Max (0.5 to 100 Hz)
	500uVrms Typ, 1mVrms Max (0.5 to 10 KHz)

Vos = Voscal +Vosexc x Vexc + Vostherm

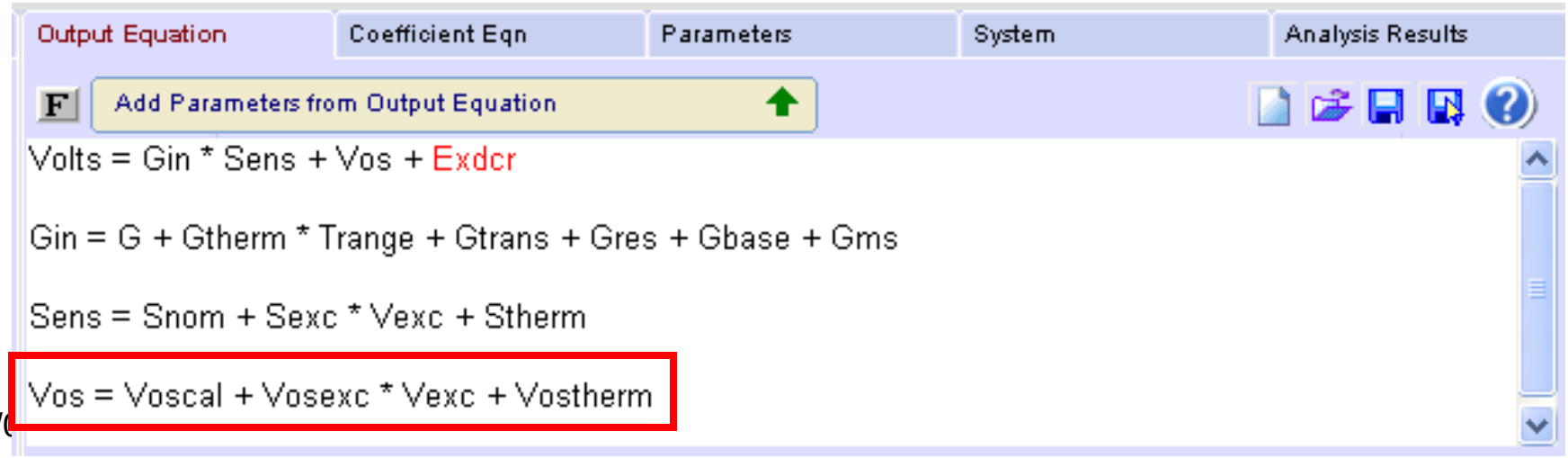
Now each of the terms in the Vos equation must be defined:

Voscal: the voltage offset obtained from the transducer lab calibration at 0 G's (V)

Vosexc: voltage offset error due to the excitation voltage (mV/V)

Vexc: the excitation applied to the accelerometer (V)

Vostherm: thermal voltage offset shift (V)



Vos = Voscal +Vosexc x Vexc + Vostherm

Voscal: the voltage offset obtained from the transducer lab calibration at 0 G's (V)

From the calibration, the zero measured output was measured to b 45.681 mV at 0 G's.

Vosexc: voltage offset error due to the excitation voltage (mV/V)

This should nominally be 0 V, and according to the spec sheet is typically 0.5 mV/V

Vexc: the excitation applied to the accelerometer (V)

As in the previous equation for sensitivity, the excitation voltage used will have a nominal value of 10V. From the TTC specification of the SCD-108S, the voltage can vary by 0.3%.

Vos = Voscal +Vosexc x Vexc + Vostherm

Vostherm: thermal voltage offset shift (V)

This should be nominally 0 V and varies 2% of the full scale output for the specified temperature range of -25ºC to 75ºC. The full scale output from the transducer lab calibration ranged from -1.945 to 2.030 V, resulting in a full scale output of 3.975 V.

Volts = Gin x Sens + Vos + Exdcr

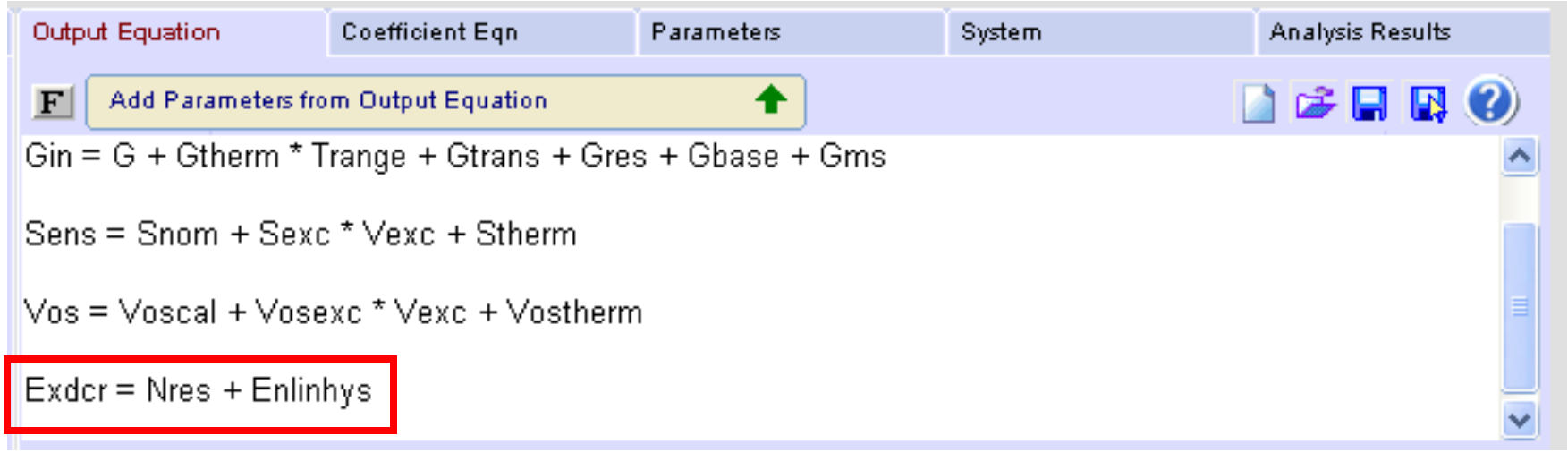
Exdcr is any other error that would contribute to the output voltage. Looking through the specifications, errors which contribute to the output voltage will be considered.

Range	±30 Gpk
Full Scale Output (FSO)	4 volts
Operating Temp Range	-55°C to 121°C
Sensitivity @100Hz	66 ±4 mV/G
Sensitivity with Excitation	+0.1%/V Typ
Offset with Excitation	-0.5mV/V Typ
Frequency Response	±5%
Mounted Resonance Freq	5500 Hz
Non-Linearity & Hysteresis	**±0.20% FSO Typ, ±0.5% FSO Max**
Transverse Sensitivity	2% Max
Zero Measurand Output	±50 mV Max
Dampening Ratio	0.7
Dampening Ratio Change	+0.08%/°C (-55°C to 121°C)
Thermal Zero Shift	±1.0% FSO Max (0-50°C)
	±2.0% FSO Max (-25-75°C)
Thermal Sensitivity Shift	±2.0% Max (0-50°C)
	±3.0% Max (-25-75°C)
Thermal Transient Error	<0.001 G/°C
Threshold (resolution)	0.008 G's
Base Strain Sensitivity	0.01 G's
Magnetic Susceptibility	<0.1 G's
Residual Noise	**100uVrms Typ, 500uVrms Max (0.5 to 100 Hz)**
	500uVrms Typ, 1mVrms Max (0.5 to 10 KHz)

Exdcr = Nres + Enlinhyst

Nres: the residual noise voltage (V)

Enlinhyst: errors due to non-linearity and hysteresis (V)



Exdcr = Nres + Enlinhyst

Nres: the residual noise voltage (V)

This quantity is nominally 0 V and varies by 0.5 mVrms. This quantity will have to be converted over to Vp, and entered into Uncertainty Analyzer as V.

(0.05 mVrms)(1.414) = 0.7071 mVp = 0.7071 mV max = 0.0007071 V

Enlinhyst: errors due to non-linearity and hysteresis (V)

Nominally 0 V, this error is 0.2% of the Full Scale Output, which was defined earlier to be 3.975 V

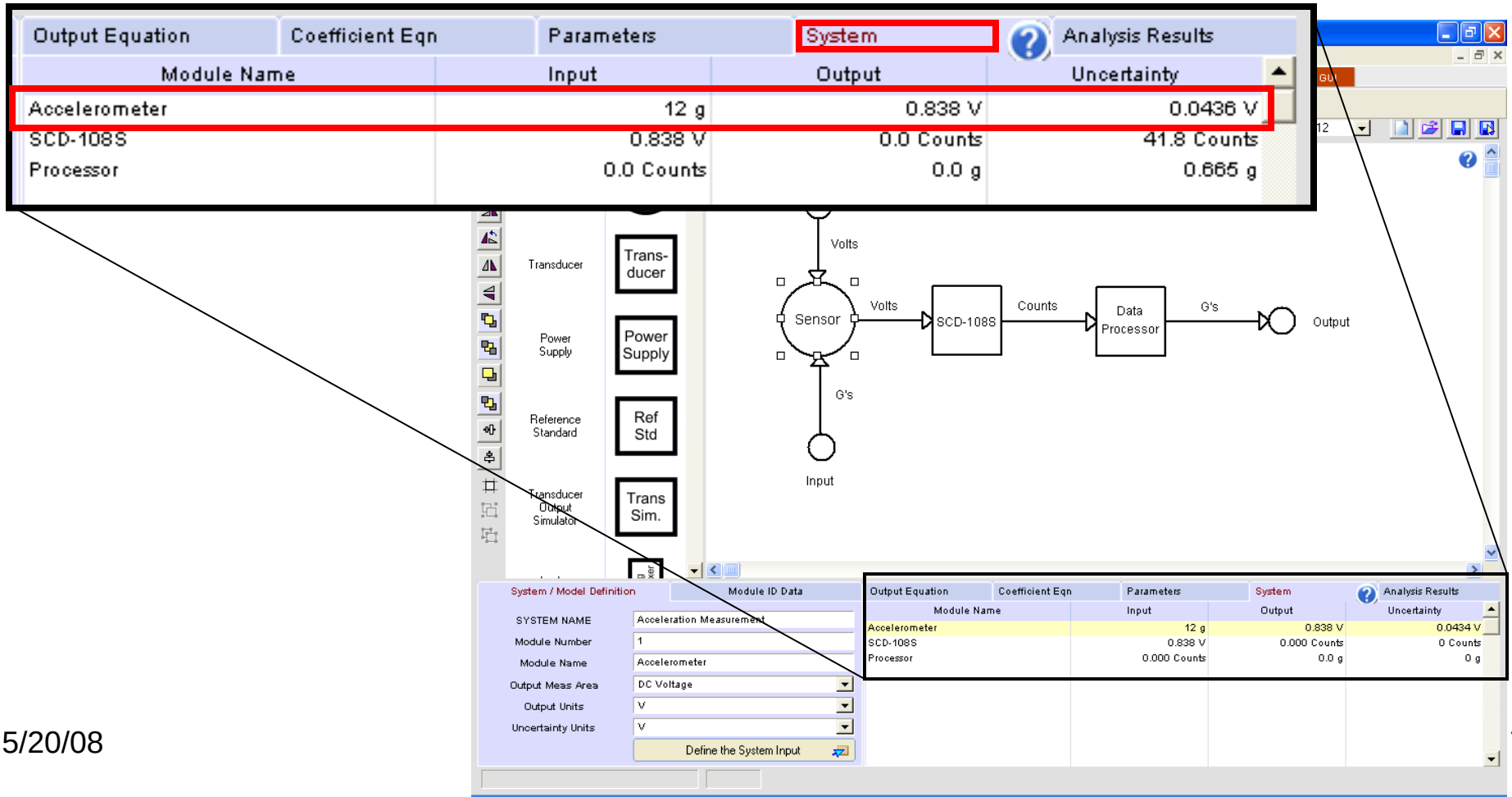
Volts = Gin x Sens + Vos + Exdcr

We are not concerned with the resonance frequency because the channel will be filtered and the dampening ratio will be ignored.

Range	±30 Gpk
Full Scale Output (FSO)	4 volts
Operating Temp Range	-55°C to 121°C
Sensitivity @100Hz	66 ±4 mV/G
Sensitivity with Excitation	+0.1%/V Typ
Offset with Excitation	-0.5mV/V Typ
Frequency Response	±5%
Mounted Resonance Freq	**5500 Hz**
Non-Linearity & Hysteresis	±0.20% FSO Typ, ±0.5% FSO Max
Transverse Sensitivity	2% Max
Zero Measurand Output	±50 mV Max
Dampening Ratio	**0.7**
Dampening Ratio Change	**+0.08%/°C (-55°C to 121°C)**
Thermal Zero Shift	±1.0% FSO Max (0-50°C)
	±2.0% FSO Max (-25-75°C)
Thermal Sensitivity Shift	±2.0% Max (0-50°C)
	±3.0% Max (-25-75°C)
Thermal Transient Error	<0.001 G/°C
Threshold (resolution)	0.008 G's
Base Strain Sensitivity	0.01 G's
Magnetic Susceptibility	<0.1 G's
Residual Noise	100uVrms Typ, 500uVrms Max (0.5 to 100 Hz)
	500uVrms Typ, 1mVrms Max (0.5 to 10 KHz)

3) Estimate the Uncertainties

Under the System tab in the System GUI, the output of the accelerometer is displayed for a given input. In this example, we chose 12 G's to be the input. From the transducer laboratory calibration, the voltage output is 0.838 V and our result matches, indicating we did not make any errors in our equations.

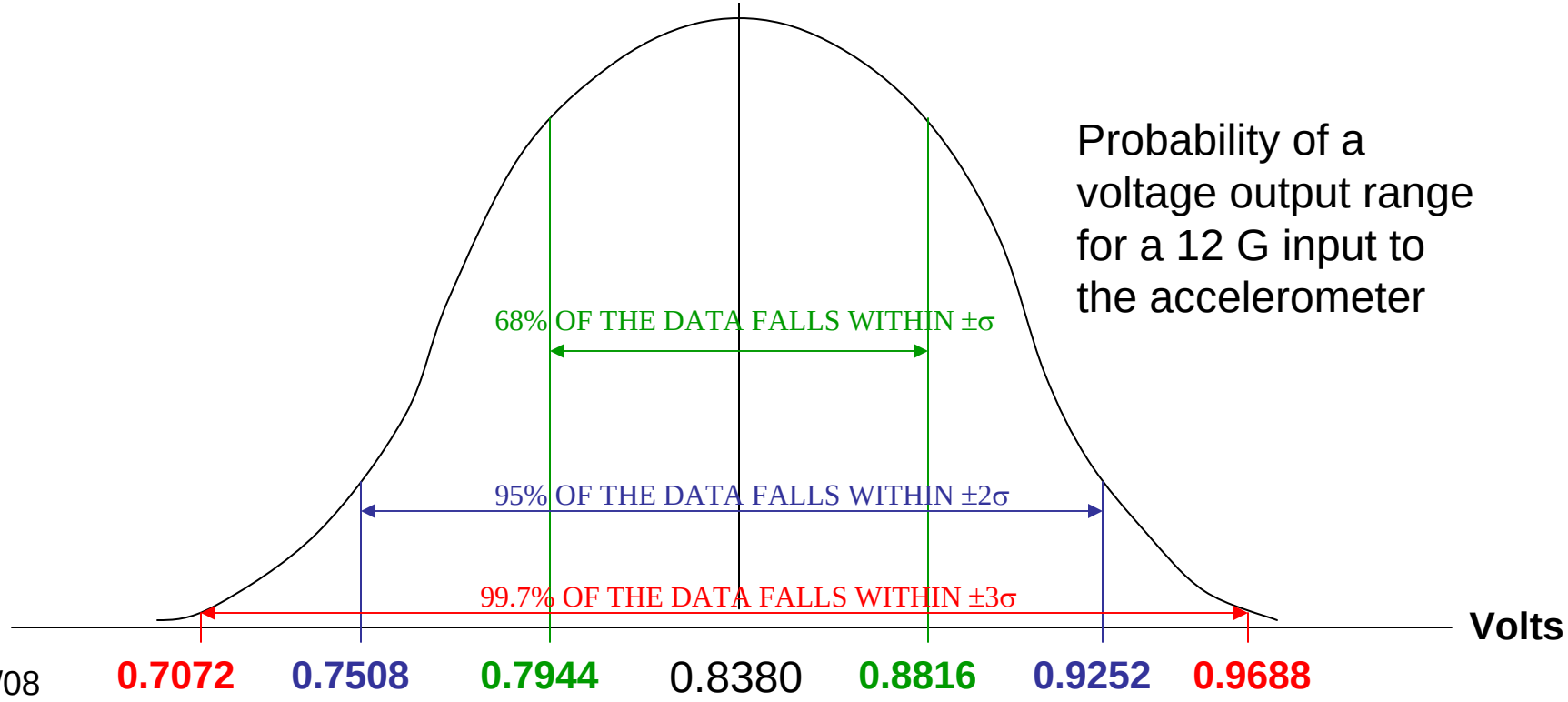


The Accelerometer Reported Uncertainty

Output Equation | Coefficient Eqn | Parameters | System | Analysis Results

Module Name	Input	Output	Uncertainty
Accelerometer	12 g	0.838 V	0.0436 V
SCD-108S	0.838 V	0.0 Counts	41.8 Counts
Processor	0.0 Counts	0.0 g	0.665 g

The reported uncertainty of 0.0436 V is the one sigma number (one standard deviation). The 95% bound would be ± two sigma, which would be ±0.0872 V. What this means is that if the accelerometer is in a 12 G (true value) normal acceleration environment, there will be a 95% confidence level that the output would be 0.838 ± 0.0872 V.



Probability of a voltage output range for a 12 G input to the accelerometer

We have just completed steps 1, 2 and 3 of the Uncertainty Analysis Procedure for the accelerometer.

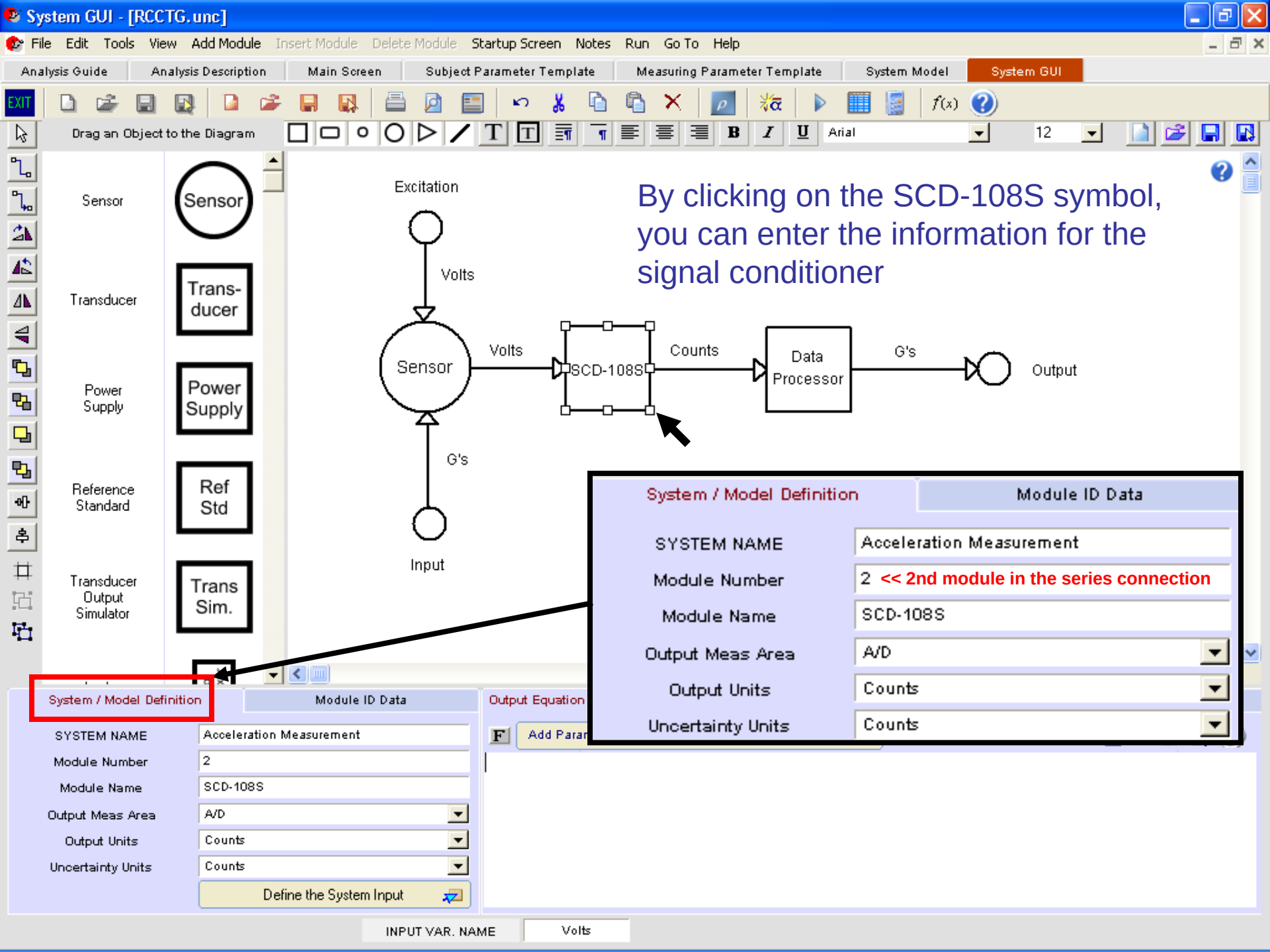
1) Define the Measurement Process
2) Identify the Error Sources and Distribution
3) Estimate Uncertainties for Each Module

As you can see, the majority of the work was determining the equations and identifying the errors along with noting any assumptions made along the way. Then, the errors were entered into the Uncertainty worksheet for each parameter.

Now we will repeat this process for the signal conditioner (SCD-108S) and the data processor.

SCD-108S Signal Conditioner

The SCD-108S signal conditioner provides gain, offset, and filtering to the input voltage. The conditioned signal is then converted it to a 12-bit digital word, and sent to the overhead MCI-105 to be inserted into the PCM stream.


System GUI - [RCCTG.unc]
File Edit Tools View Add Module Insert Module Delete Module Startup Screen Notes Run Go To Help
Analysis Guide
Analysis Description
Main Screen
Subject Parameter Template
Measuring Parameter Template
System Model
System GUI
Drag an Object to the Diagram
Sensor
Transducer
Power Supply
Reference Standard
Transducer Output Simulator
Excitation
Volts
Sensor
Volts
SCD-108S
Counts
Data Processor
G's
Output
G's
Input
By clicking on the SCD-108S symbol, you can enter the information for the signal conditioner
System / Model Definition
Module ID Data
SYSTEM NAME
Acceleration Measurement
Module Number
2 << 2nd module in the series connection
Module Name
SCD-108S
Output Meas Area
A/D
Output Units
Counts
Uncertainty Units
Counts
Output Equation
Define the System Input
INPUT VAR. NAME
Volts

SCD-108S Signal Conditioner

TTC's Specifications for the SCD-108S

Excitation Error: 0.3% (used in the accelerometer analysis)

Gain Error: 0.4%

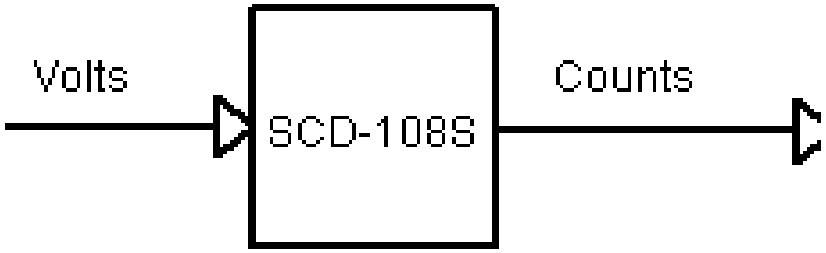
Offset Error:

For Primary Gain	Offset Error
1	±10mV
10	±10mV
100	±15mV
1000	±60mV

This data is assumed to be a channel-to-channel error of the gain and offset. For example, randomly picking a card and channel with a gain of 1, we would expect a 95% chance of the gain falling between 0.96 and 1.04.

If we were performing an on-aircraft substitution calibration, we would use the information from the calibration to determine the actual gain and offset of the channel.

SCD-108S Signal Conditioner



$$\text{Counts} = (\text{C1scd})(\text{VoltsCond}) + \text{C0scd}$$

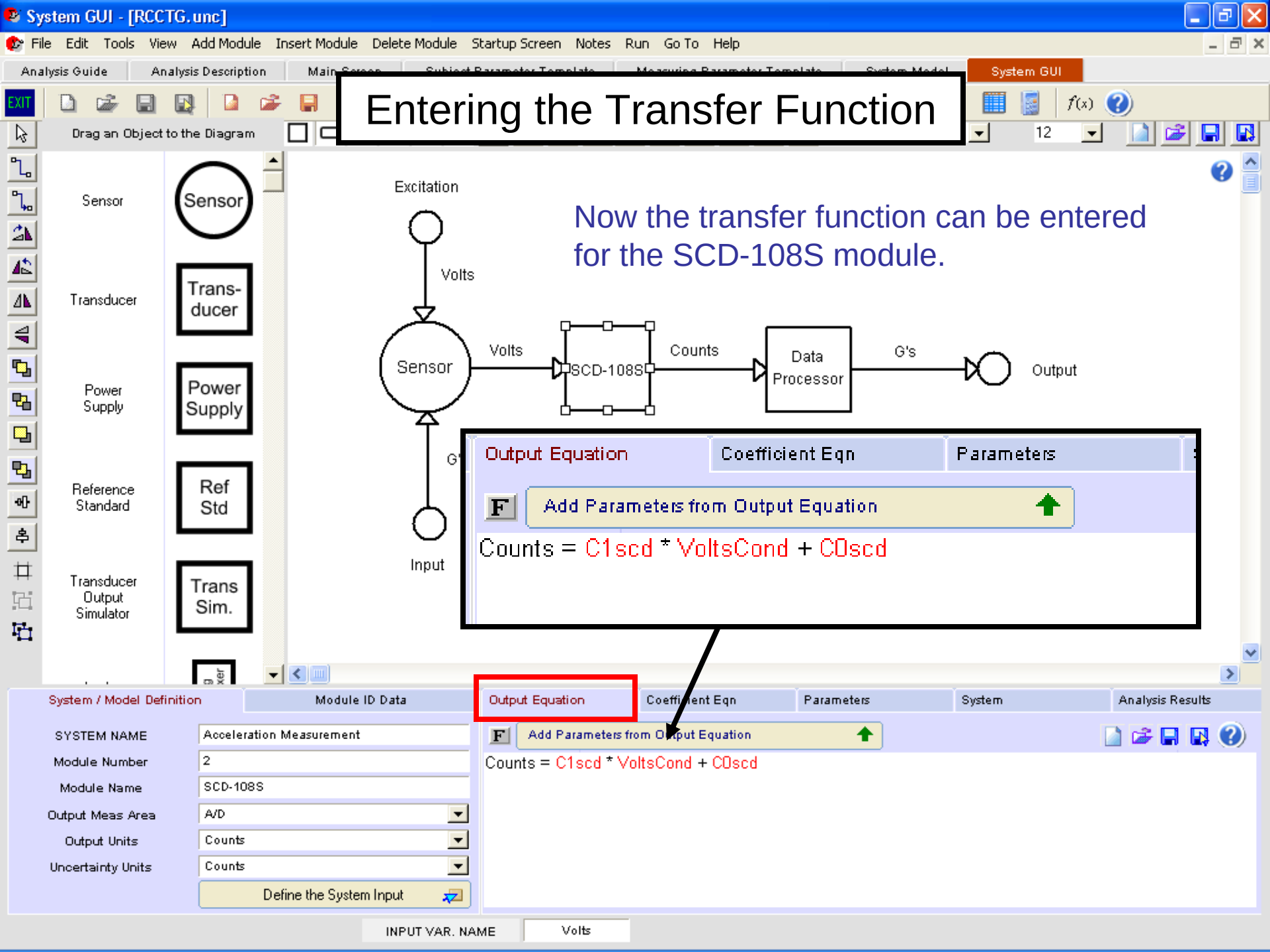
C1scd: the C1 coefficient for the 12-bit analog-to-digital converter (Counts/V)

VoltsCond: the input voltage to the analog-to-digital converter (V)

C0scd: the C0 coefficient for the 12-bit analog-to-digital converter (Counts)

Entering the Transfer Function

Now the transfer function can be entered for the SCD-108S module.



Counts = C1scd x VoltsCond + C0scd

C1scd: the C1 coefficient for the 12-bit analog-to-digital converter (Counts/V)
C0scd: the C0 coefficient for the 12-bit analog-to-digital converter (Counts)

The input range into the ADC on the SCD card is ±5 volts.

5 V = 4048 counts
-5 V = 48 counts

Therefore, the ADC equation converting volts to counts is

Counts = C1sdc x volts + C0scd

Counts = 400 x volts + 2048

C1scd = 400 and C0scd = 2048

These coefficients will be considered to be constants and not have any error. The error in quantization will be included in the VoltsCond analysis.

Counts = C1scd x VoltsCond + C0scd

VoltsCond: the conditioned input voltage to the analog-to-digital converter (V)

The conditioned voltage will be defined by the following equation:

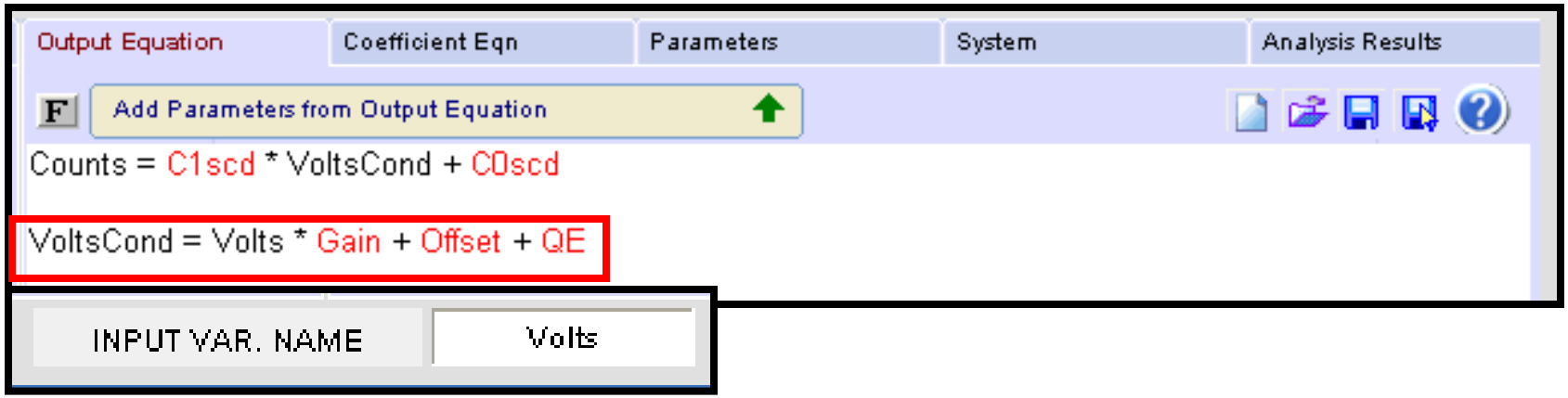
VoltsCond = Volts x Gain + Offset + QE

Volts: the voltage output from the accelerometer (V)

Gain: the programmed gain on the SCD-108S (V/V)

Offset: the programmed offset on the SCD-108S (V)

QE: the quantization error in the ADC (V)



System Model - [RCCTG.unc]

File Edit View Tools Startup Screen Run Notes Go To Help

Analysis Guide | Analysis Description | Main Screen | Subject Parameter Template | Measuring Parameter Template | System Model | System GUI

Procedure

- Define the System Input
- Identify the System and Module
- Set the Measurement Area and Units
- Enter the Module Output Equation
- Define the Module Parameters
- Establish the Correlations
- Update the Notes
- Select Report Options
- Print the Module Report
- Update the Module Library

System / Module Definition | Module ID Data

Module Output Equation

Counts = C1scd * VoltsCond + C0scd

VoltsCond = Volts * Gain + Offset + QE

Click “Add Parameters” to have them listed and then each error can be defined in the Uncertainty Worksheet.

Module Parameters

Add Parameters from Output Equation

Edit	Description	Parameter Name	Standard Uncertainty	% Confidence	± Error Limits	Coefficient	Component Uncertainty (Counts)
	SCD C1 Coefficient	C1scd		95.00		2.0	
	SCD Offset Coefficient	C0scd		95.00		1.0	
	Amplifier Gain	Gain	0.004880 V/V	95.00	0.0096	335.3	1.636090
	SCD Offset	Offset	0.0051 V	95.00	0.0100	400.0	2.0409
	Quantization Error	QE	0.00076 V	95.00	0.0015	400.0	0.30387
	Module Input Variable	Volts	0.0434 V	-	-	956	41.4904

Insert Module | Delete Module

41.6 Counts

SYSTEM ANALYSIS

Module Name	Input
Accelerometer	
SCD-108S	

Update

	12 g
	2,849.7 Counts
	41.6 Counts
	95.00 %
	1.9600
	Infinite
	82 Counts

Edit	Description	Parameter Name
	SCD C1 Coefficient	C1scd
	SCD Offset Coefficient	C0scd
	Amplifier Gain	Gain
		Offset
		QE

Error Source Uncertainty Worksheet
Digital Sampling Error Worksheet
Computation Error Worksheet

Module 2 | SCD-108S

VoltsCond = Volts x Gain + Offset + QE

Volts: the voltage output from the accelerometer (V)

Gain: the programmed gain on the SCD-108S (V/V)

The programmed gain for this channel is 2.3911 V/V. The specification states that there is a gain error of 0.4% of the nominal gain.

Offset: the programmed offset on the SCD-108S (V)

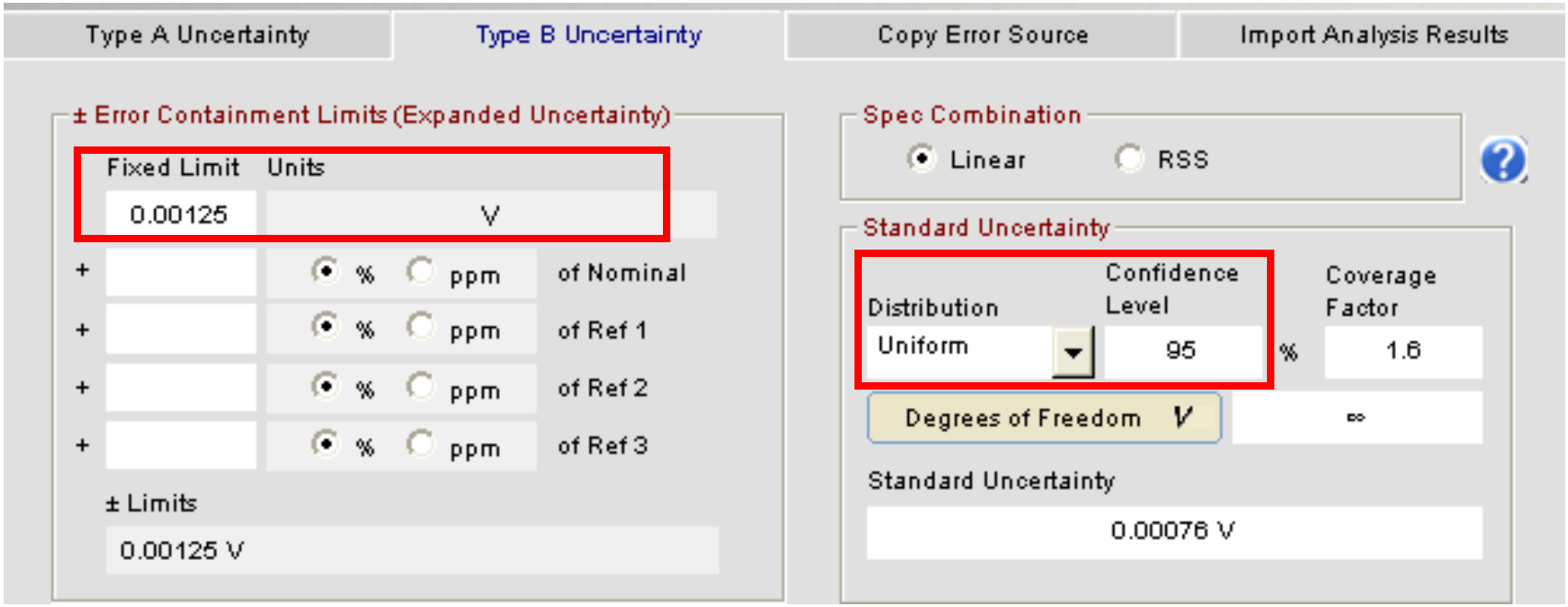
The programmed offset was selected to be 0 V. For the primary gain selection of 1, the offset error is stated to be ±10mV, or 0.010 V

All of these parameters are entered like the previous ones.

VoltsCond = Volts x Gain + Offset + QE

QE: the quantization error in the ADC (V)

The quantization error is defined to be one half of the volts per count sensitivity
QE = ±½(volts/count)
QE = ±½(1/400)
QE = ±½(0.0025) = ±0.00125 V
Quantization errors are assumed to have a uniform distribution

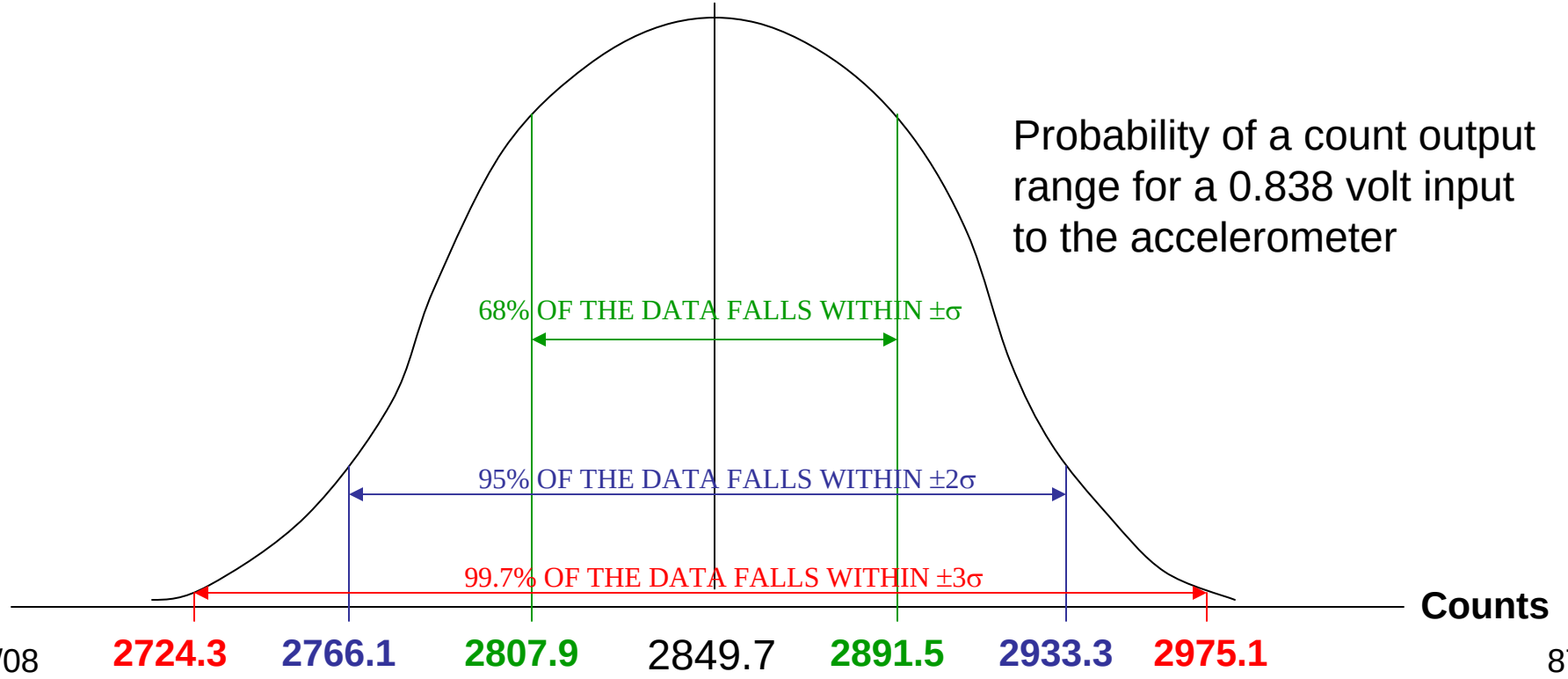


The SCD-108S Reported Uncertainty

Output Equation | Coefficient Eqn | Parameters | System | Analysis Results

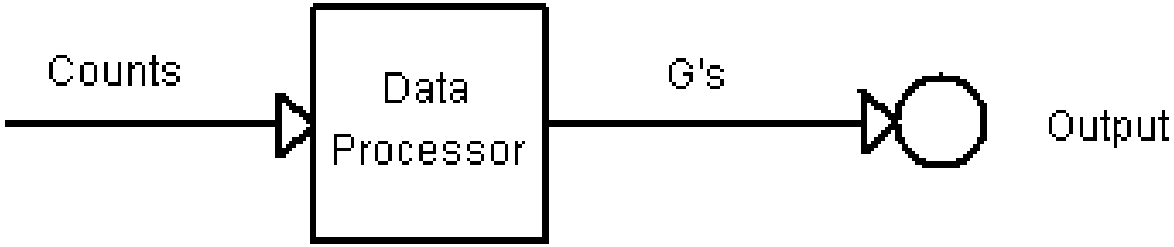
Module Name	Input	Output	Uncertainty
Accelerometer	12 g	0.838 V	0.0436 V
SCD-108S	0.838 V	2,849.7 Counts	41.8 Counts
Processor	2,849.7 Counts		

The reported uncertainty of 41.8 counts is the one sigma number (one standard deviation). The 95% bound would be ± two sigma, which would be ±83.6 counts. This can be interpreted as: for an input voltage of 0.838 V, there will be a 95% confidence level that the output would be 2849.7 ± 83.6 counts.



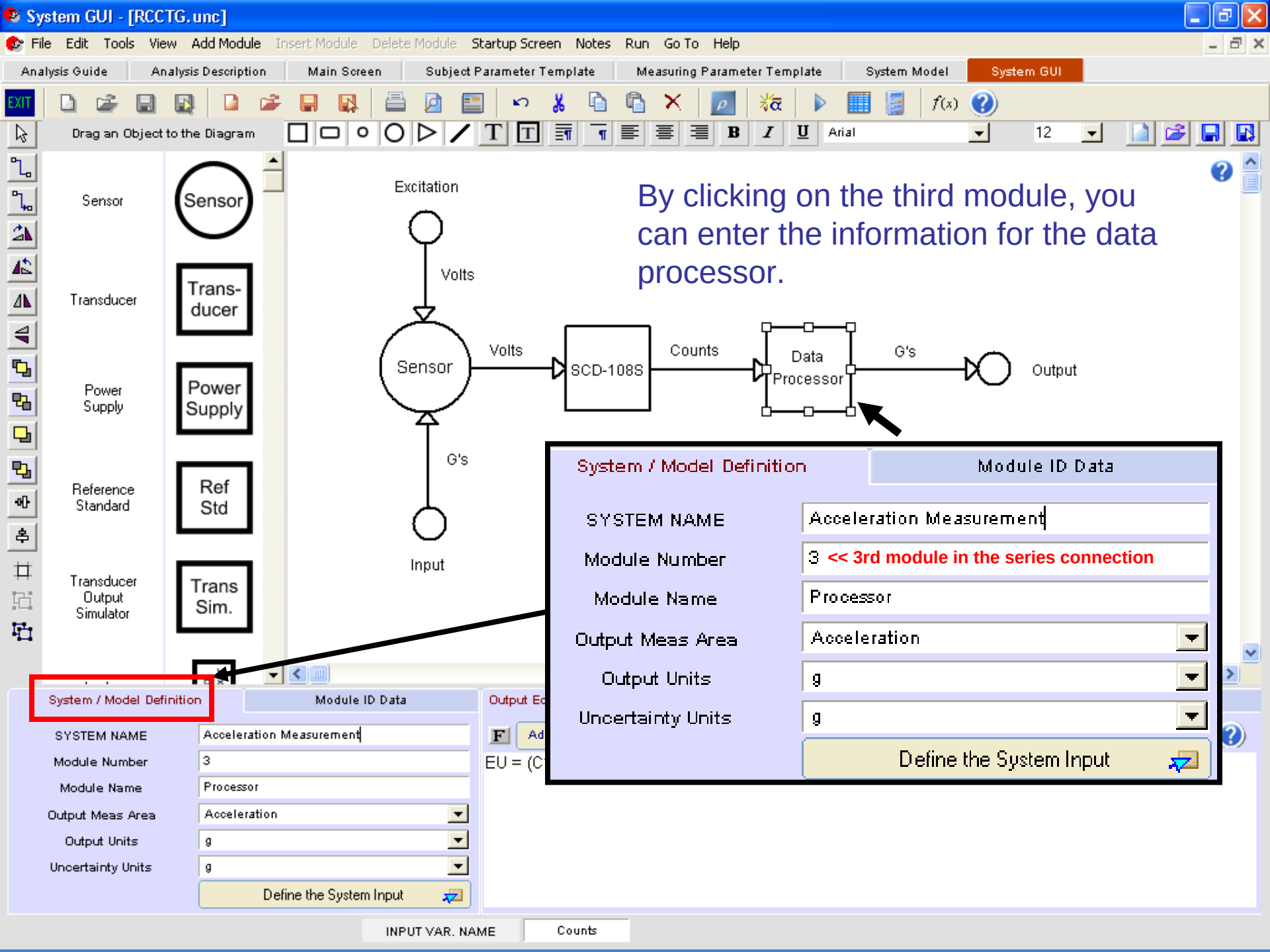
Probability of a count output range for a 0.838 volt input to the accelerometer

The Data Processor



The data processor portion is the process in which the counts get converted back to engineering units by the ground station.

The order of going from G’s to counts is then reversed, so we must come up with the equation to output the acceleration in G’s.


System GUI - [RCCTG.unc]
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Analysis Description
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System GUI
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Power Supply
Reference Standard
Transducer Output Simulator
Excitation
Volts
Sensor
Volts
SCD-108S
Counts
Data Processor
G's
Output
G's
Input
By clicking on the third module, you can enter the information for the data processor.
System / Model Definition
Module ID Data
SYSTEM NAME
Acceleration Measurement
Module Number
3 << 3rd module in the series connection
Module Name
Processor
Output Meas Area
Acceleration
Output Units
g
Uncertainty Units
g
Define the System Input
INPUT VAR. NAME
Counts

The Data Processor EU Conversion

From Transducer Laboratory Calibration :
EU = C1 x Volts + C0 + Res

Res is the residual error in the laboratory calibration (G)

C1= 15.12179 and C0 = -0.6819215
Rearranging the equation, Volts = (EU – C0 – Res) / C1

From the SCD-108S transfer function:
Counts = C1scd(Volts x Gain + Offset) + C0scd
Counts = C1scd([(EU – C0 – Res) / C1] x Gain + Offset) + C0scd

Solving this equation for EU, you get
EU = (C1/Gain) x [(Counts – C0scd)/C1scd – Offset] + C0 + Res

EU = (C1/Gain) x [(Counts – C0scd)/C1scd – Offset] + C0 + Res

C0: C0 coefficient from the transducer lab calibration (G)

C1: C1 coefficient from the transducer lab calibration (G/V)

Res: Residual error from the transducer lab calibration (G)

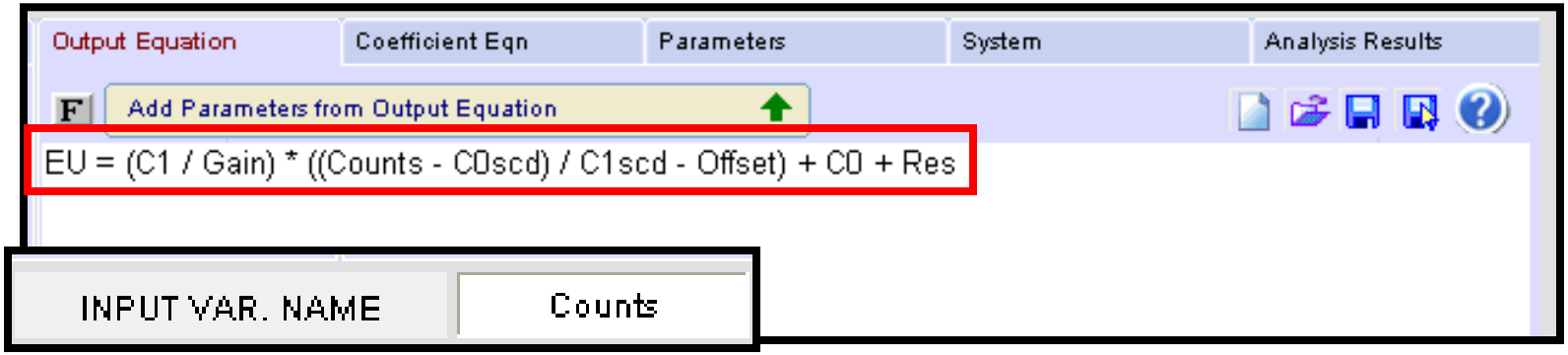
Counts: the input to the processor module (Counts)

C1scd: the calculated C1 coefficient from the signal conditioner (Counts/V)

C0scd: the calculated C0 coefficient from the signal conditioner (Counts)

Gain: the programmed gain in the signal conditioner (V/V)

Offset: the programmed offset in the signal conditioner (V)



EU = (C1/Gain) x [(Counts – C0scd)/C1scd – Offset] + C0 + Res

C0: C0 coefficient from the transducer lab calibration (G)
C1: C1 coefficient from the transducer lab calibration (G/V)

The transducer lab provided a LSBF equation relating G's to volts
G's = 15.12179 x Volts – 0.6819215
C0 = -0.681925 and C1 = 15.12179
These are constants with no variations

Res: Residual error from the transducer lab calibration (G)

From the transducer laboratory calibration, the average residual error would be 0, with a standard deviation of 0.0662. The 95% bound would be twice that or ±0.1324.

Counts: the input to the processor module (Counts)

"Counts" is the output of the data system, so the uncertainties will be determined by the output of that module.

$$EU = (C1/Gain) \times [(Counts - C0scd)/C1scd - Offset] + C0 + Res$$

C1scd: the calculated C1 coefficient from the signal conditioner (Counts/V)
C0scd: the calculated C0 coefficient from the signal conditioner (Counts)

These are constant values that we defined earlier.
C1scd = 400 Counts/V and C0scd = 2048 Counts

Gain: the programmed gain in the signal conditioner (V/V)

The programmed gain for this channel is 2.3911 V/V. The specification stated that there was a gain error of 0.4% of the nominal gain.

Offset: the programmed offset in the signal conditioner (V)

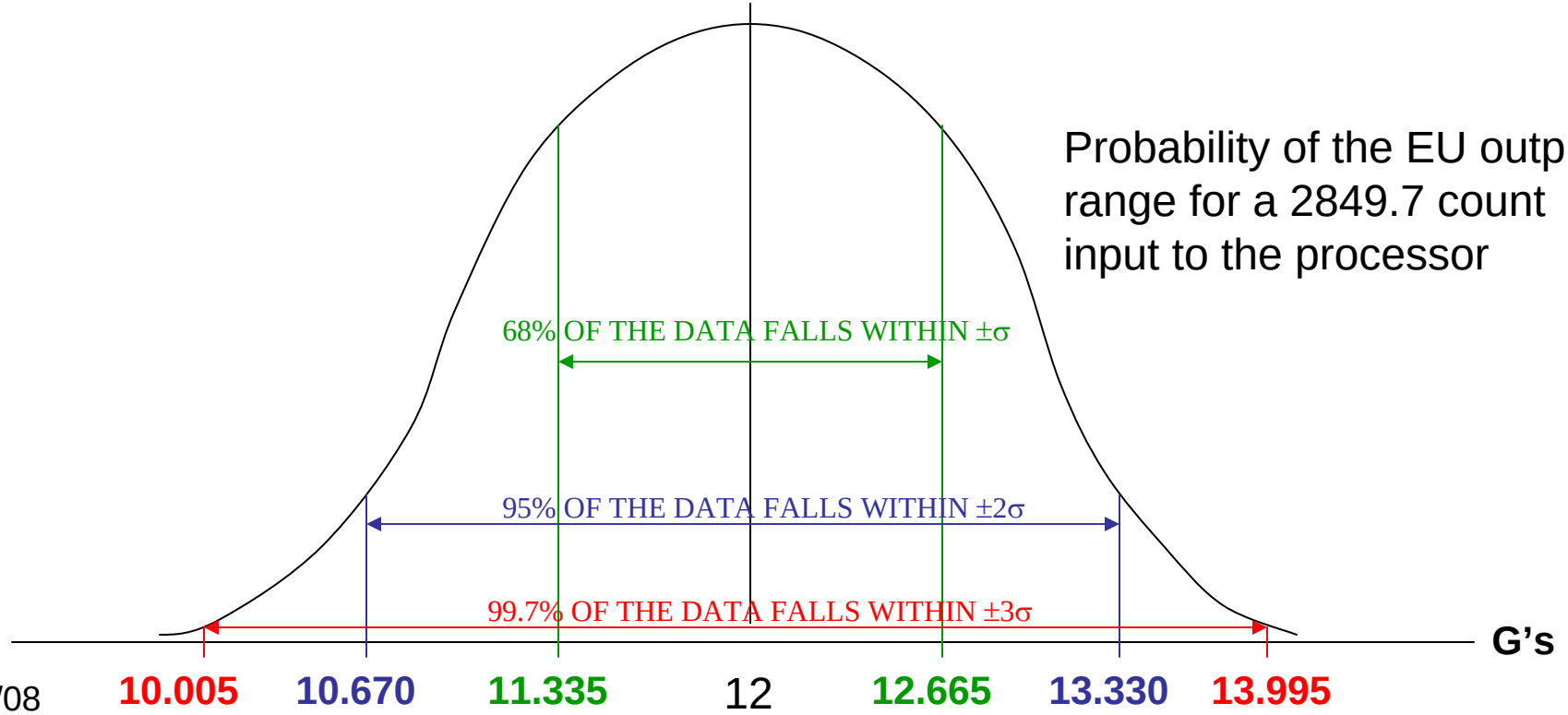
The programmed offset was selected to be 0 V. For the primary gain selection of 1, the offset error was stated to be ±10mV, or ±0.010 V

The Processor Reported Uncertainty

Output Equation | Coefficient Eqn | Parameters | System | Analysis Results

Module Name	Input	Output	Uncertainty
Accelerometer	12 g	0.838 V	0.0436 V
SCD-108S	0.838 V	2,849.7 Counts	41.8 Counts
Processor	2,849.7 Counts	12.0 g	0.665 g

The reported uncertainty of 0.665 G's is the one sigma number (one standard deviation). The 95% bound would be ± two sigma, which would be ±1.33 G's. What this means is that if the processor receives a count value of 2849.7 Counts, there will be a 95% confidence level that the output would be 12 ± 1.33 G's.



Probability of the EU output range for a 2849.7 count input to the processor

Steps 1 through 3 of the uncertainty analysis procedure are now complete for each module. We now have the uncertainty that each module contributes to the measurement system.

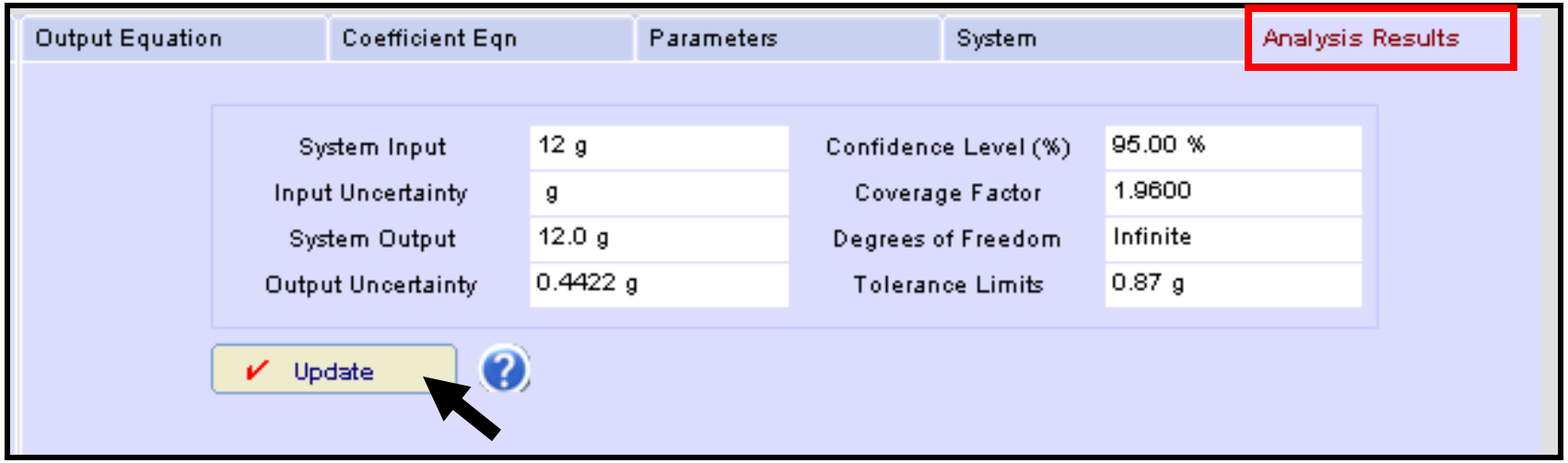
We will now proceed to step 4 of the procedure, combining uncertainties.

4) Combine Uncertainties (of All Modules)

To compute the overall uncertainty of the system, select the Analysis Results tab in the System GUI Screen. Click on “Update” button to incorporate any changes you might have made.

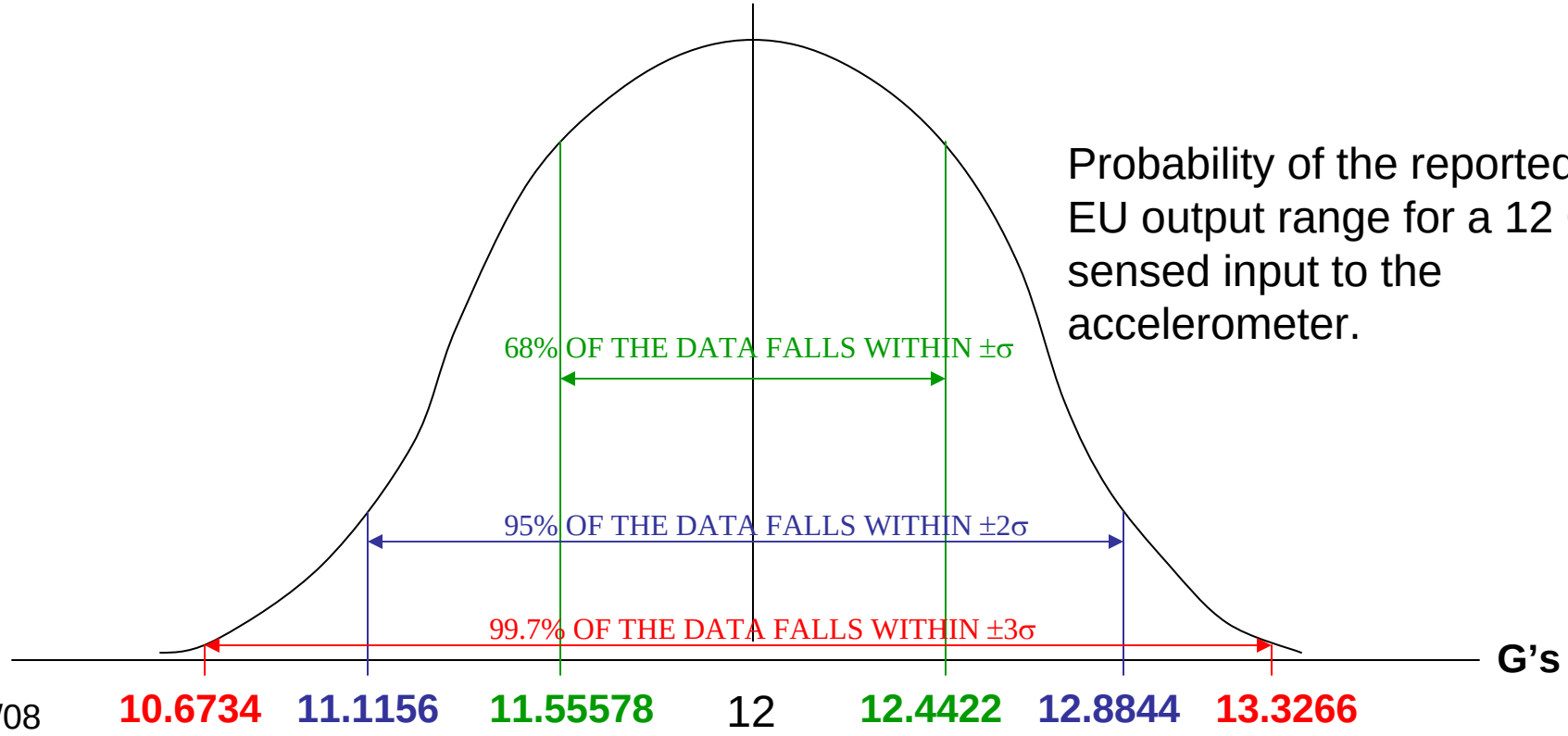
The results will not match the output of the data processor shown in the previous slide. The previous slide indicated the uncertainties for each individual module. The Analysis Results is giving you the uncertainty for the entire measurement system.

The overall system uncertainty takes into account the errors from one module feeding into the following module.



System Input	12 g	Confidence Level (%)	95.00 %
Input Uncertainty	g	Coverage Factor	1.9600
System Output	12.0 g	Degrees of Freedom	Infinite
Output Uncertainty	0.4422 g	Tolerance Limits	0.87 g

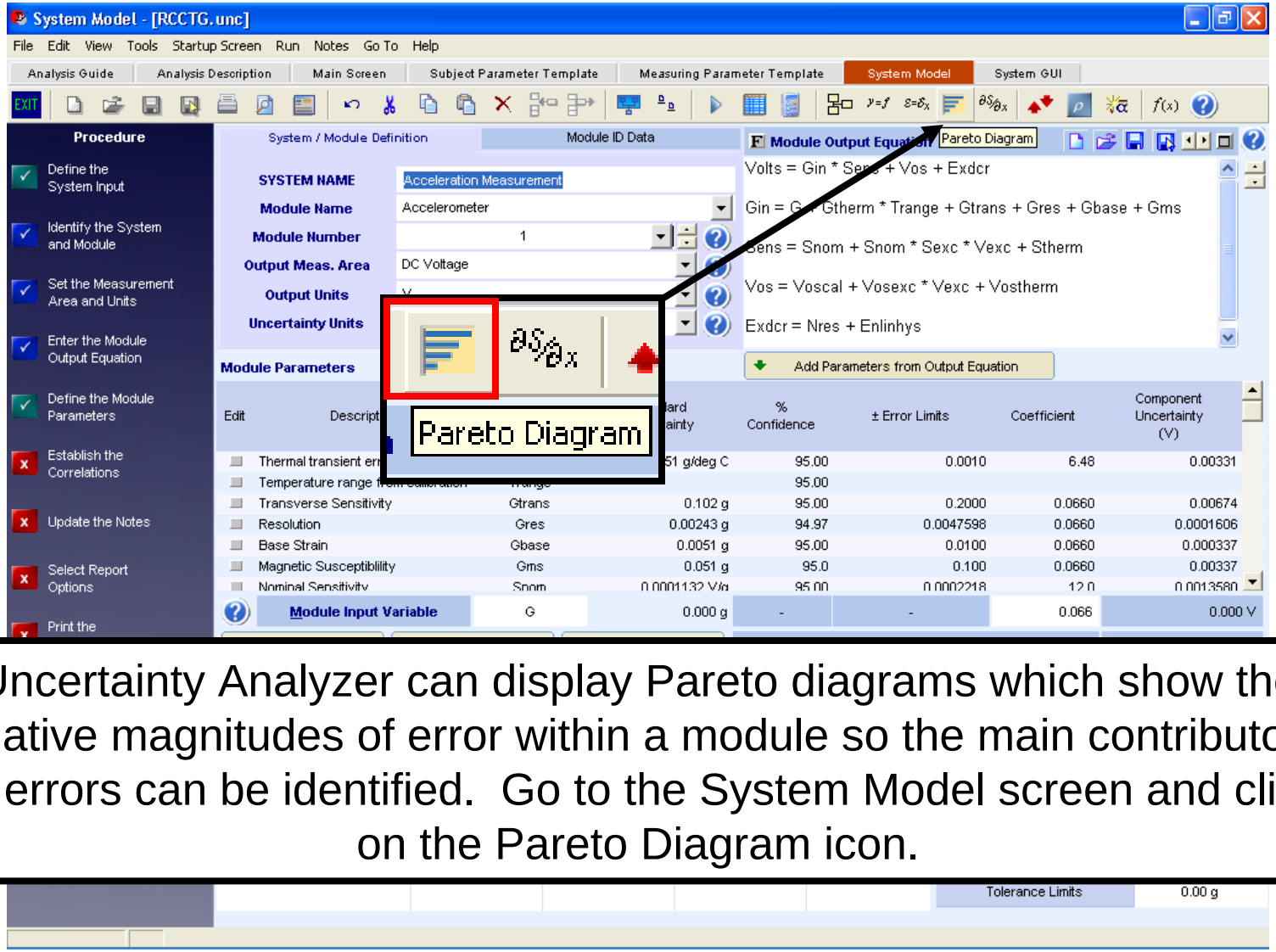
The output uncertainty of the system is 0.4422 G's and the 2 sigma 95% bound would be 12 ± 0.87 G's.



Probability of the reported EU output range for a 12 G sensed input to the accelerometer.

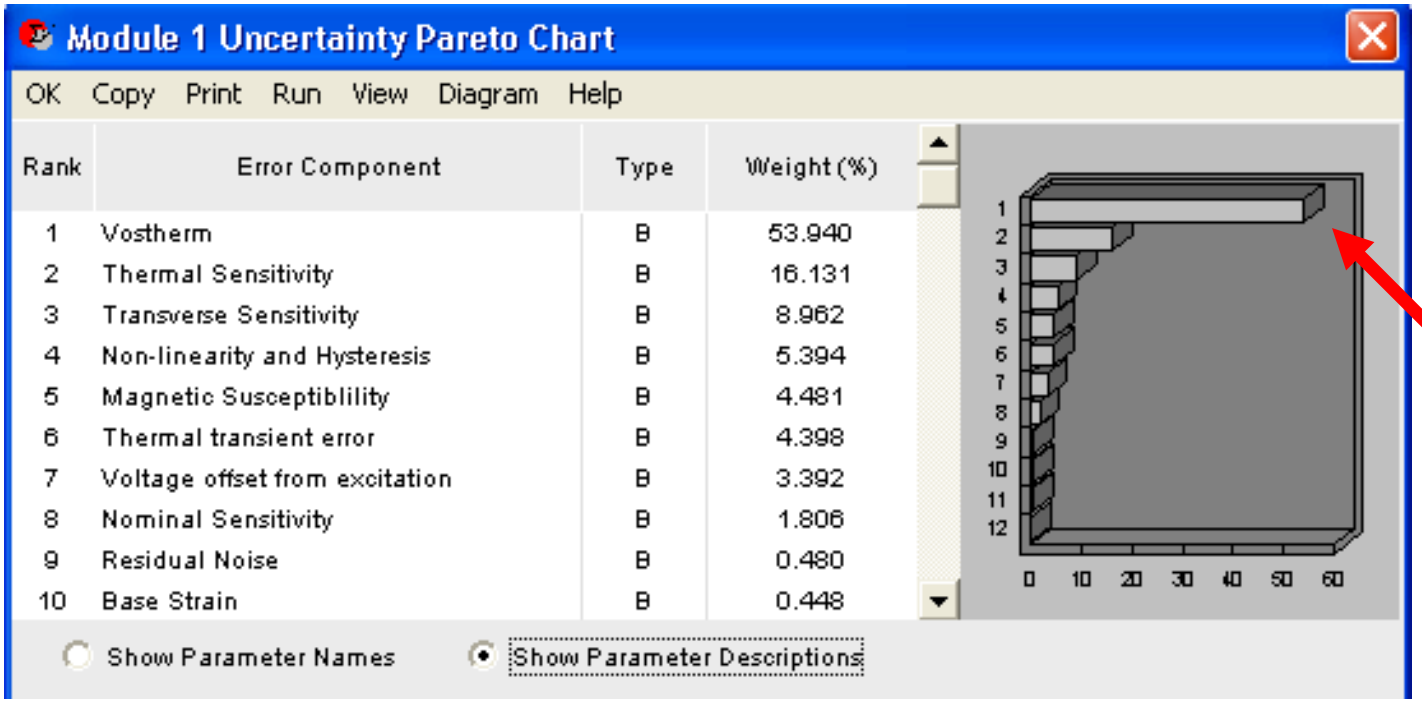
5) Report the Analysis Results

Finding the Main Contributors of Error



Uncertainty Analyzer can display Pareto diagrams which show the relative magnitudes of error within a module so the main contributors of errors can be identified. Go to the System Model screen and click on the Pareto Diagram icon.

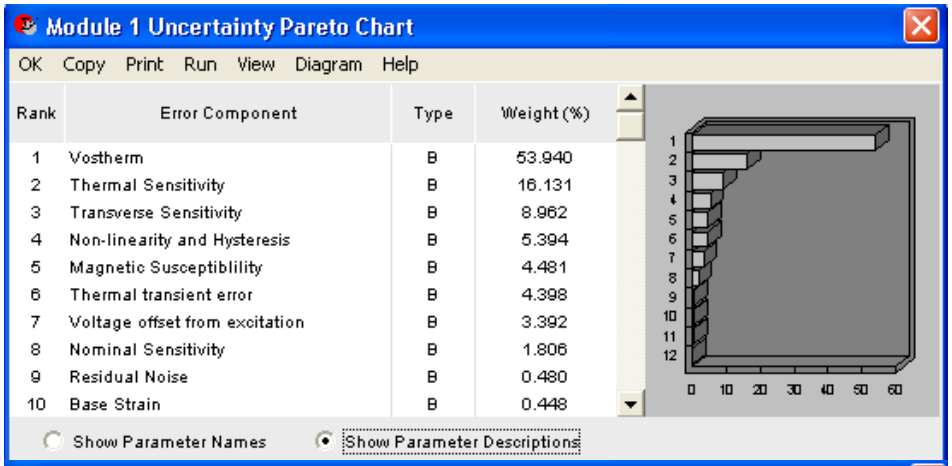
When displaying the Pareto Diagrams, the main contributors of the error in a module really stand out.



Rank	Error Component	Type	Weight (%)
1	Vostherm	B	53.940
2	Thermal Sensitivity	B	16.131
3	Transverse Sensitivity	B	8.962
4	Non-linearity and Hysteresis	B	5.394
5	Magnetic Susceptiblility	B	4.481
6	Thermal transient error	B	4.398
7	Voltage offset from excitation	B	3.392
8	Nominal Sensitivity	B	1.806
9	Residual Noise	B	0.480
10	Base Strain	B	0.448

Main contributor of error

Comparing the Pareto diagrams of the three modules in our measurement system, the source of the main contributor of error can be very easily identified.



Rank	Error Component	Type	Weight (%)
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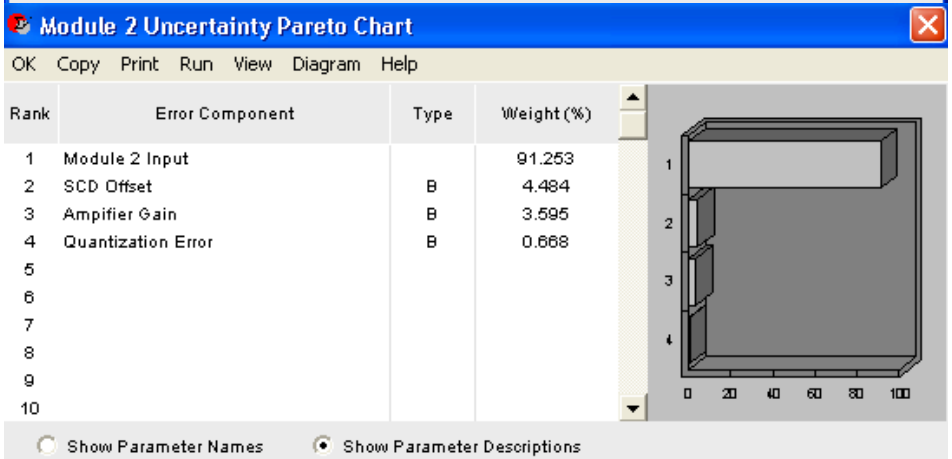
7	Voltage offset from excitation	B	3.392
8	Nominal Sensitivity	B	1.806
9	Residual Noise	B	0.480
10	Base Strain	B	0.448

Module 1: Accelerometer

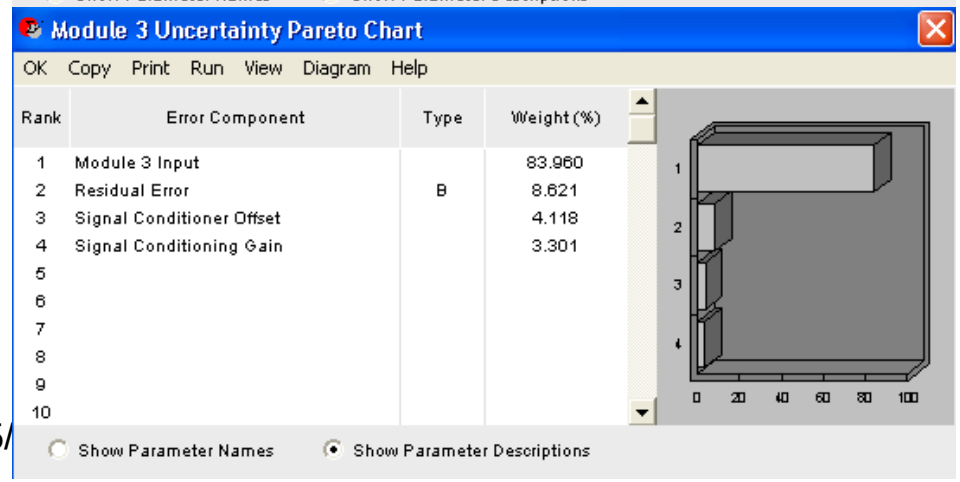
Voltage offset due to thermal effects (53.94%)



Rank	Error Component	Type	Weight (%)
1	Module 2 Input		91.253
2	SCD Offset	B	4.484
3	Amplifier Gain	B	3.595
4	Quantization Error	B	0.668
5			
6			
7			
8			
9			
10			

Module 2: Signal Conditioner

Input (module 1 output) (91.25%)



Rank	Error Component	Type	Weight (%)
1	Module 3 Input		83.960
2	Residual Error	B	8.621
3	Signal Conditioner Offset		4.118
4	Signal Conditioning Gain		3.301
5			
6			
7			
8			
9			
10			

Module 3: Data Processor

Input (module 2 output) (83.95%)

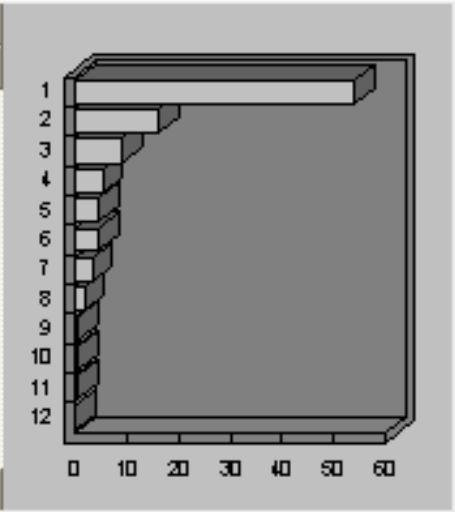
Finding the Main Contributors of Error

From the Pareto diagrams, the main contributor of the error in the accelerometer really stood out. Now that the error is identified, what can be done to improve this measurement?

Module 1 Uncertainty Pareto Chart

OK Copy Print Run View Diagram Help

Rank	Error Component	Type	Weight (%)
1	Vostherm	B	53.940
2	Thermal Sensitivity	B	16.131
3	Transverse Sensitivity	B	8.962
4	Non-linearity and Hysteresis	B	5.394
5	Magnetic Susceptiblility	B	4.481
6	Thermal transient error	B	4.398
7	Voltage offset from excitation	B	3.392
8	Nominal Sensitivity	B	1.806
9	Residual Noise	B	0.480
10	Base Strain	B	0.448



Show Parameter Names Show Parameter Descriptions

The accelerometer's main contributor (53.9%) of error is the offset due to thermal effects. Coming in second is the thermal sensitivity at 16.13%. To help improve the uncertainty of the overall measurement, either the temperature range we assumed is too wide, or this accelerometer may not be the right one for this measurement environment.

Purpose of the Uncertainty Analysis

In flight test, an uncertainty analysis is done in order to see if an accuracy request can be met. Hopefully the person doing the requesting is knowledgeable in what is being requested.

The results of the uncertainty analysis determines if the components used do in fact meet the requirements.

The advantage of using the Uncertainty Analyzer software is that you can change some of the assumptions and see how it effects the outcome, or you can easily replace the accelerometer module with another model or manufacturer.

Modifying the Uncertainty Analysis

With the use of Uncertainty Analyzer it is very easy to adjust your assumptions. Because *Vostherm* was the main contributor to our error, we might question if the temperature range chosen was correct.

Vostherm: thermal voltage offset shift (V)

This should be nominally 0 V and varies 2% of the full scale output for the specified temperature range of -25ºC to 75ºC. The full scale output from the transducer lab calibration ranged from -1.945 to 2.030 V, resulting in a full scale output of 3.975 V.

Say for example, it is later noted that the accelerometer is mounted in the cabin of the aircraft.

Using the 1% FSO error in the accelerometer specs for the temperature range of 0ºC to 50ºC would be more realistic for the accelerometer located in the cabin. This change would be made within the Vostherm error source uncertainty worksheet.

Modifying the Uncertainty Analysis

The next highest contributing error, is *Stherm*, the thermal sensitivity shift. Because this was also dependent on the assumed temperature range, it will be updated also.

Stherm: thermal sensitivity shift (V/G)

For the temperature range of -25 to 75ºC, the error is 3% of the nominal sensitivity.

Using the error of 2% of the nominal sensitivity for the temperature range of 0ºC to 50ºC would be more realistic for the accelerometer located in the cabin. This change is made within the Stherm error source uncertainty worksheet.

Observing the Results of the Temperature Range Change in the Accelerometer

Module 1 Uncertainty Pareto Chart — **First Analysis**

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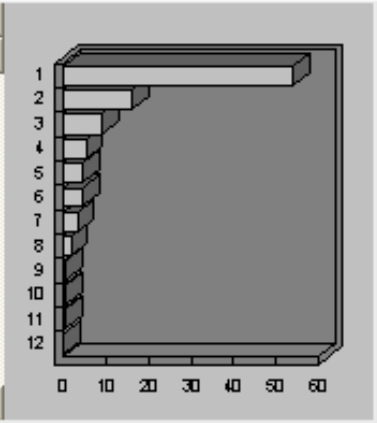
Rank	Error Component	Type	Weight (%)
1	Vostherm	B	53.940
2	Thermal Sensitivity	B	16.131
3	Transverse Sensitivity	B	8.962
4	Non-linearity and Hysteresis	B	5.394
5	Magnetic Susceptiblility	B	4.481
6	Thermal transient error	B	4.398
7	Voltage offset from excitation	B	3.392
8	Nominal Sensitivity	B	1.806
9	Residual Noise	B	0.480
10	Base Strain	B	0.448



Show Parameter Names Show Parameter Descriptions

Module 1 Uncertainty Pareto Chart — **Second Analysis**

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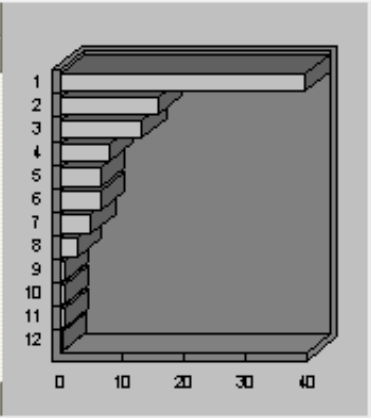
Rank	Error Component	Type	Weight (%)
1	Vostherm	B	39.865
2	Thermal Sensitivity	B	15.896
3	Transverse Sensitivity	B	13.246
4	Non-linearity and Hysteresis	B	7.973
5	Magnetic Susceptiblility	B	6.623
6	Thermal transient error	B	6.501
7	Voltage offset from excitation	B	5.014
8	Nominal Sensitivity	B	2.669
9	Residual Noise	B	0.709
10	Base Strain	B	0.662



Show Parameter Names Show Parameter Descriptions

Observing the Results of the Temperature Range Change On the System

First Analysis

System Input	12 g	Confidence Level (%)	95.00 %
Input Uncertainty	g	Coverage Factor	1.9600
System Output	12.0 g	Degrees of Freedom	Infinite
Output Uncertainty	0.4422 g	Tolerance Limits	0.87 g

Second Analysis

System Input	12 g	Confidence Level (%)	95.00 %
Input Uncertainty	g	Coverage Factor	1.9600
System Output	12.0 g	Degrees of Freedom	Infinite
Output Uncertainty	0.14213 g	Tolerance Limits	0.28 g

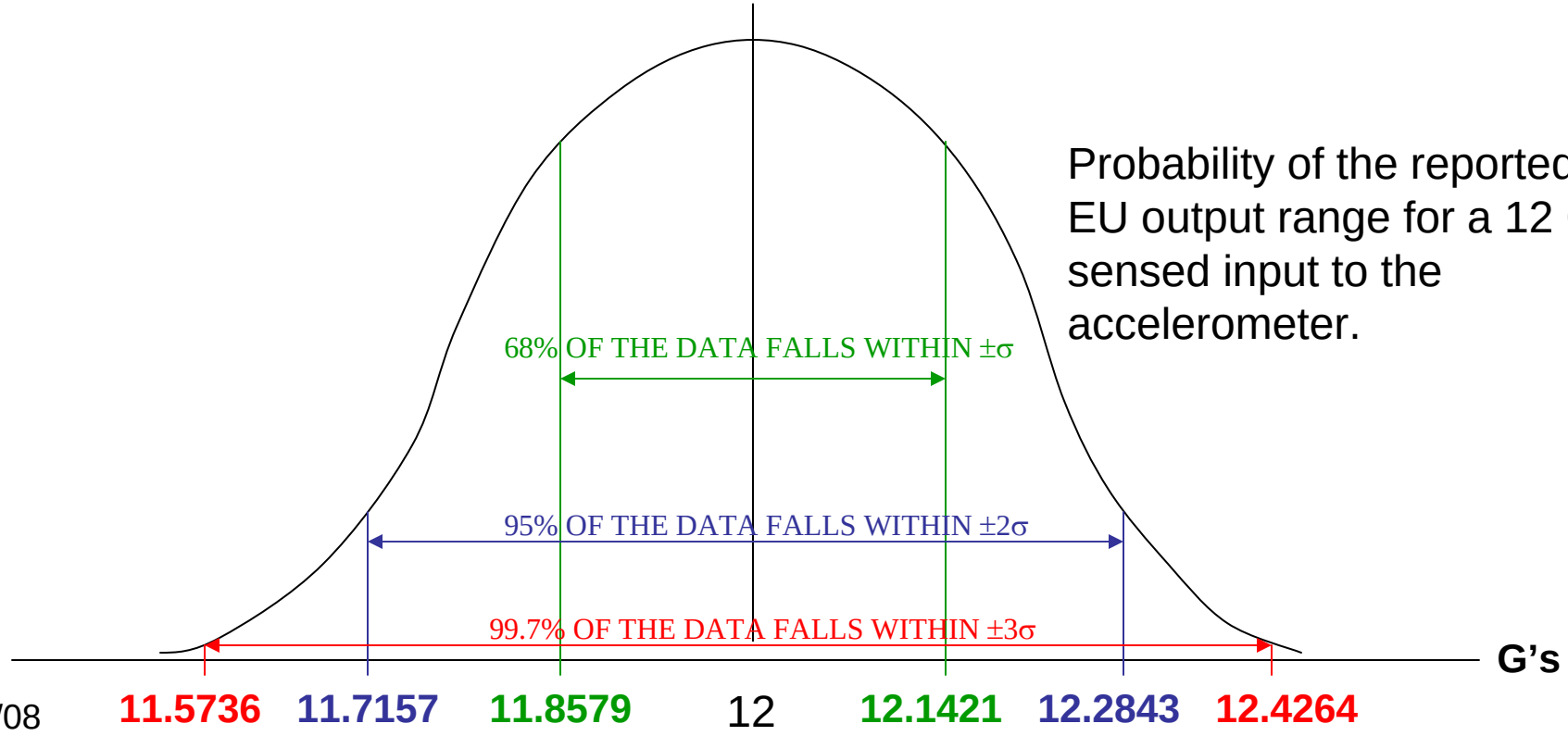
Being able to reduce the Vostherm error in half, and the Stherm by a third, resulted in reducing the overall system 2σ tolerance limits from 0.87 to 0.28 G's.

If the accelerometer was to be mounted in a more extreme temperature range, then another accelerometer would have to be used to get better results.

Combine Uncertainties of All Modules

System Input	12 g	Confidence Level (%)	95.00 %
Input Uncertainty	g	Coverage Factor	1.9600
System Output	12.0 g	Degrees of Freedom	Infinite
Output Uncertainty	0.14213 g	Tolerance Limits	0.28 g

Redoing the tolerance limits of the measurement, the output uncertainty of the system is 0.1421 G's and the 2 sigma 95% bound would be 12 ± 0.28 G's.



Probability of the reported EU output range for a 12 G sensed input to the accelerometer.

Reporting Uncertainty to the Customer

When reporting your uncertainty to the person requesting the analysis, be sure to explain what your results mean.

If the customer will be comparing your results to another analysis, be sure to have them understand that:

1) Your reported uncertainty is $n\sigma$ (n standard deviations) and that a certain percent of the data collected will fall within that range of the true value – but that true value is never known.

2) If the customer will be comparing your results to another analysis, have them note the confidence intervals and the assumptions of both analyses.

3) This may have to be done in a face-to-face meeting to have them fully understand the information you are providing to them.

Uncertainty Analyses

- By following the procedure covered in this presentation, just about any measurement can be analyzed.

- The difficult part is defining the errors present in the system, determining how they relate to the other errors, and making the assumptions about the environment that the measurement is exposed to.

- The Uncertainty Analyzer software easily identifies the large errors in the Pareto diagram.

Uncertainty Analyses

- If the results of the analysis do not meet the measurement uncertainty requirement, the main contributing errors can be analyzed to see if bad assumptions were made or the equipment cannot meet the measurement requirements.

- Changing the assumptions can easily be made within the software to see their impact on the overall uncertainty.

- Replacing an entire module can be done without too much difficulty.

References

- ISG Uncertainty Analyzer 3.0 User Manual
- ISG Uncertainty Training Course Student Workbook
- IRIG-122-07 Uncertainty Analysis Principles and Methods
- Measurement Uncertainty – Methods and Applications, Ronald H. Dieck
- Experimentation and Uncertainty Analysis for Engineers, Coleman & Steele

Are there any questions ?